Sequential decisions under uncertainty
Markov Decision Processes (MDP)

Tomáš Svoboda

Vision for Robots and Autonomous Systems, Center for Machine Perception
Department of Cybernetics
Faculty of Electrical Engineering, Czech Technical University in Prague

April 1, 2020

slyším velmi dobře
ším nic moc, ale dostatečně 1
nedostatečně 0

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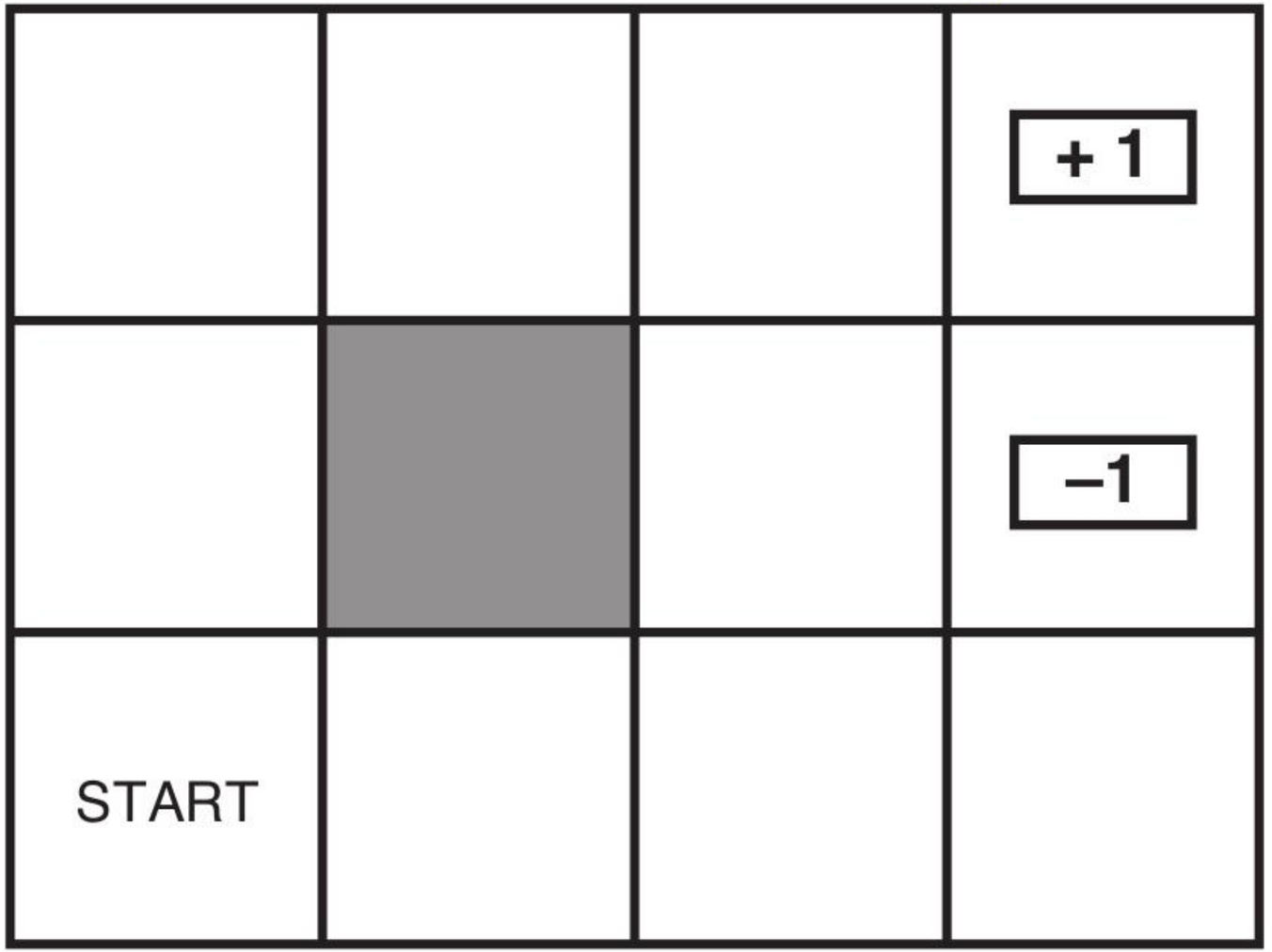
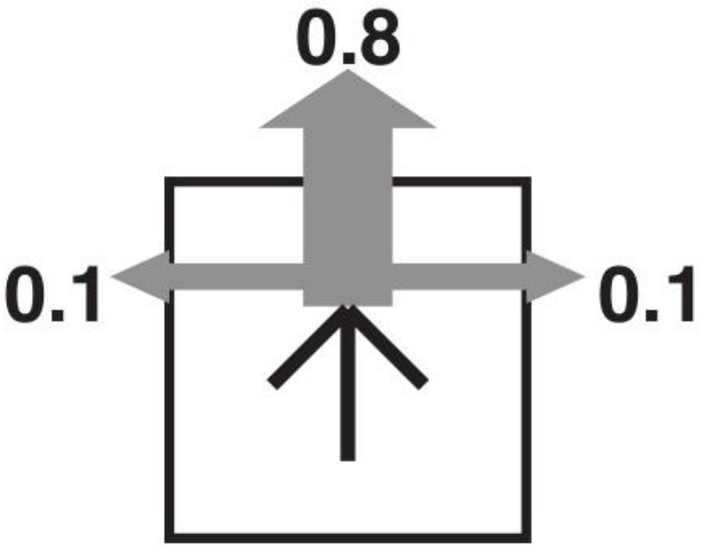
Unreliable actions in observable grid world





States $s \in S$, actions $a \in A$

(Transition) Model $T(s, a, s') \equiv p(s'|s, a) =$ probability that a in s leads to s'

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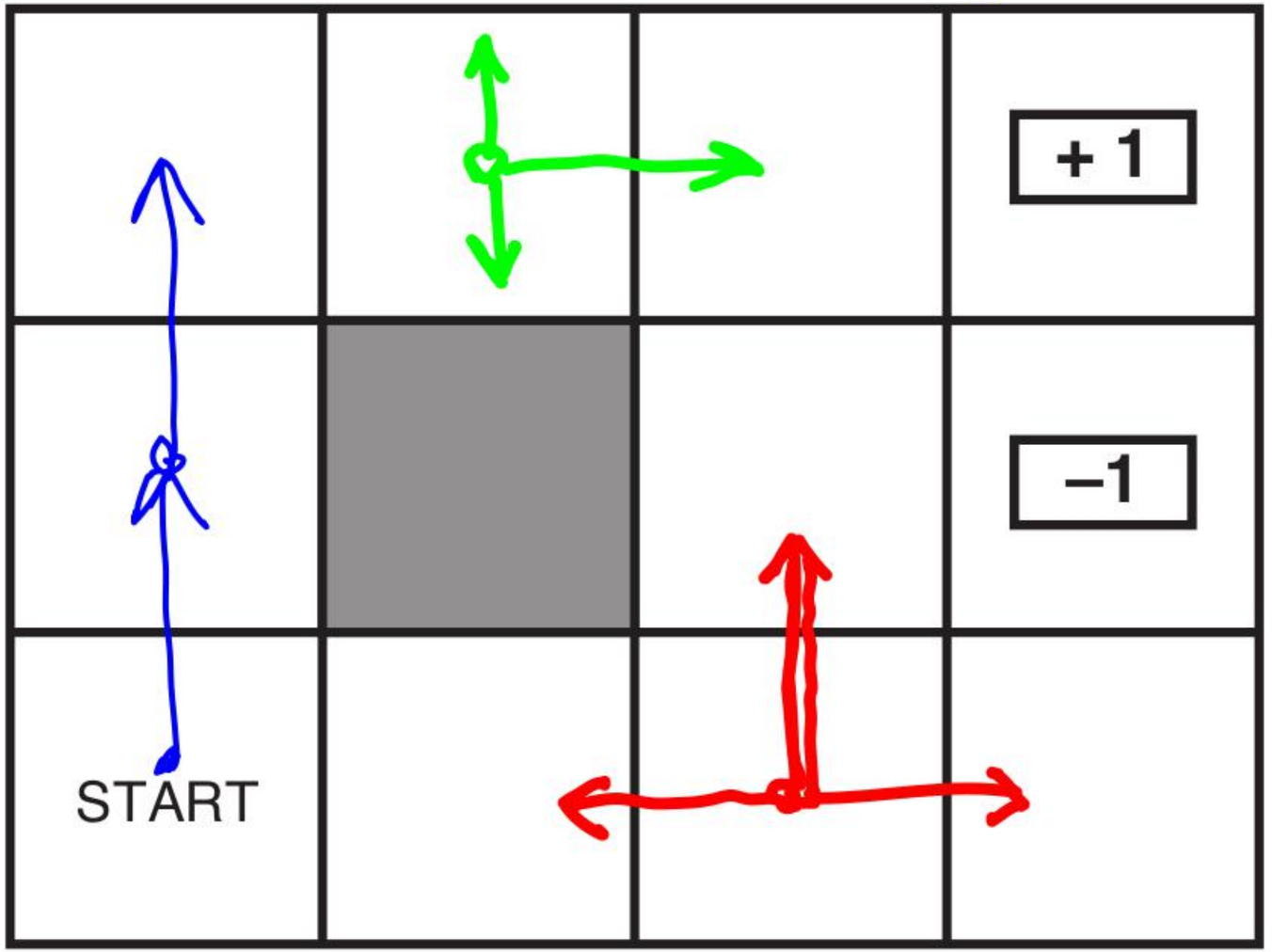

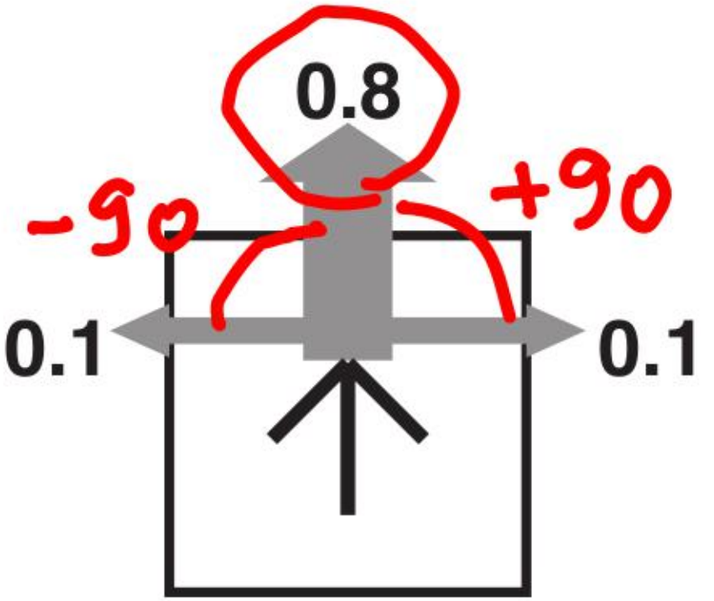

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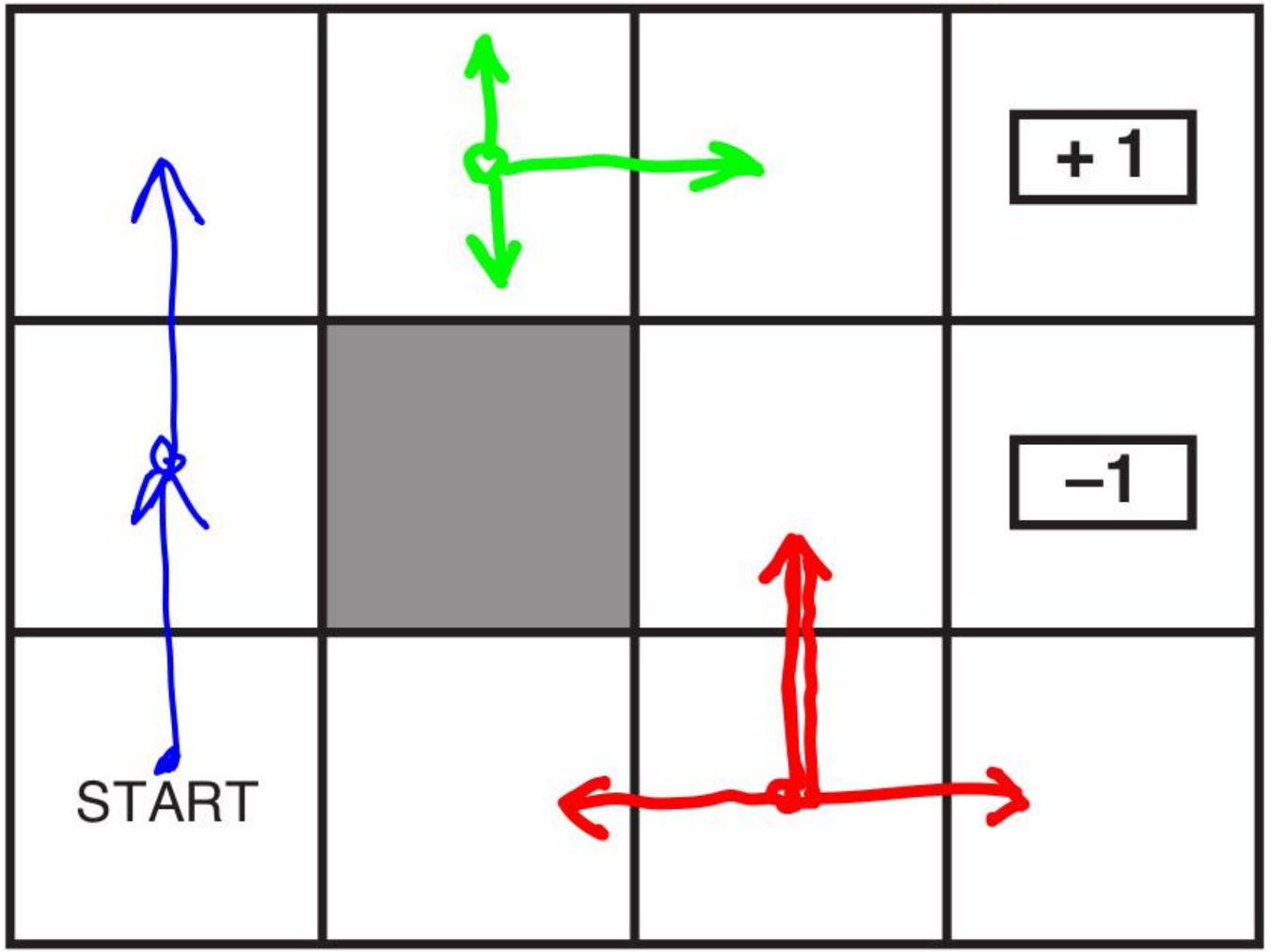

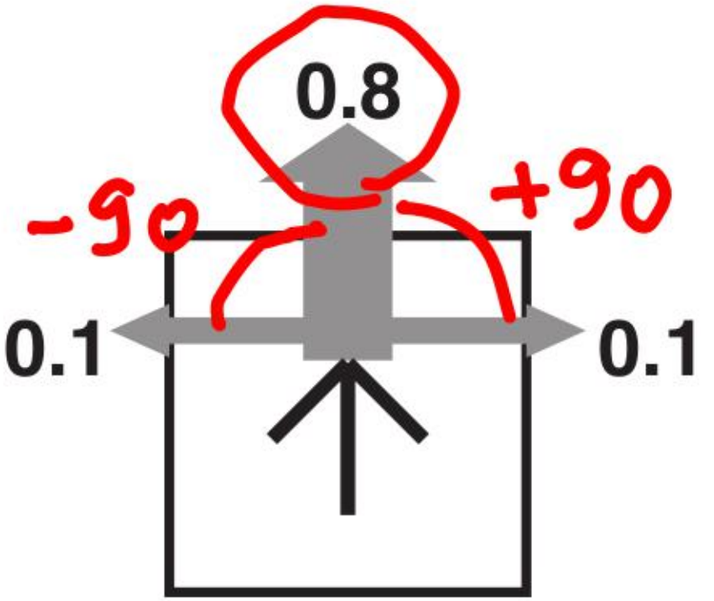

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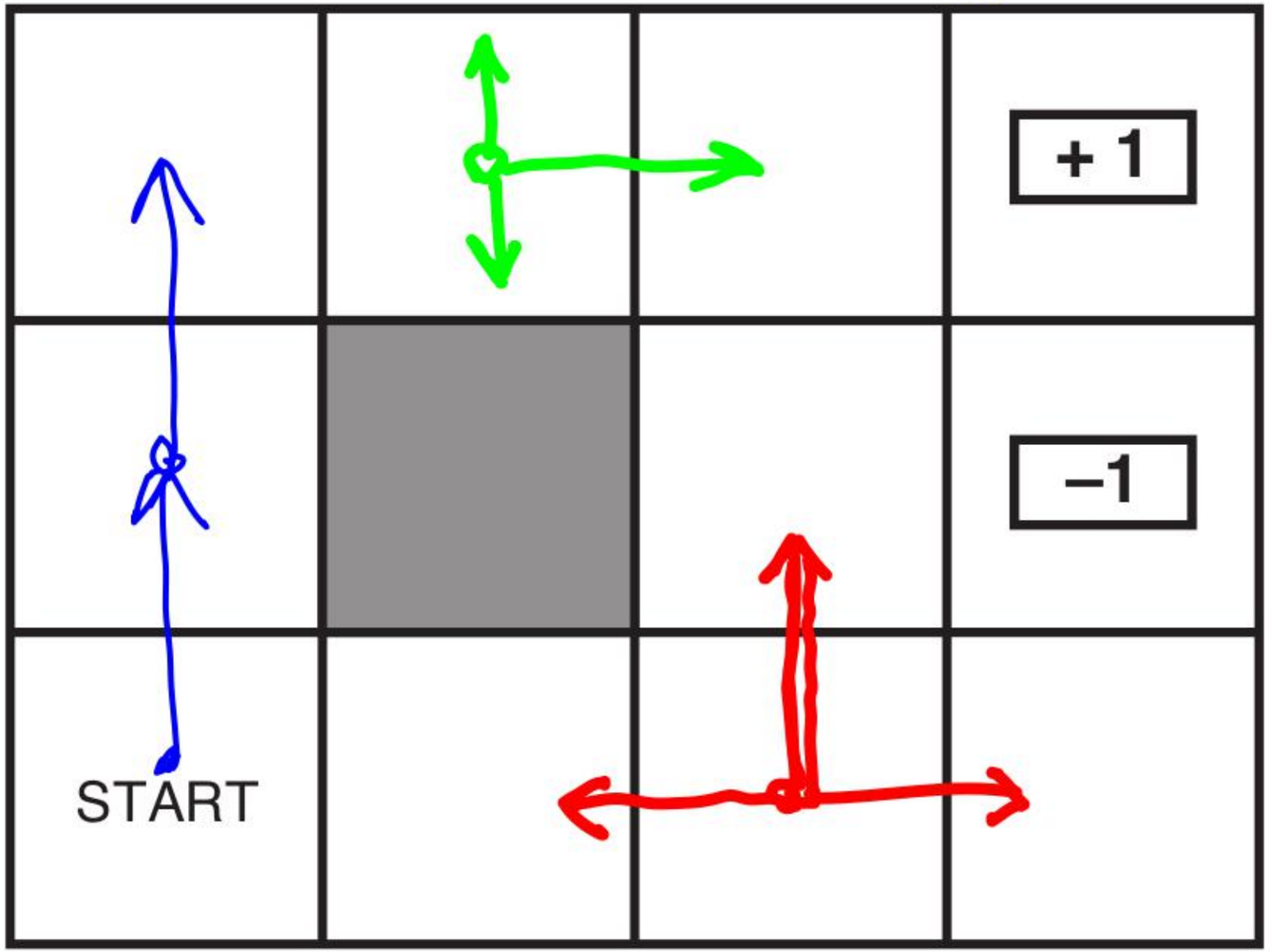

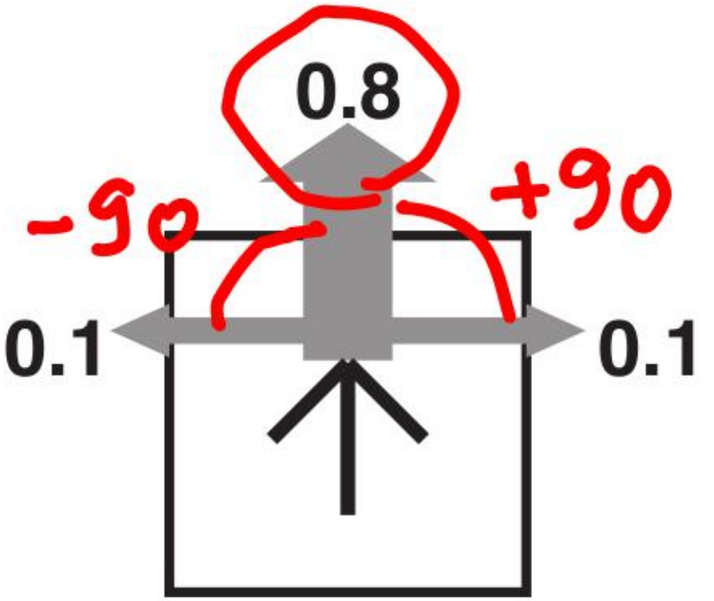

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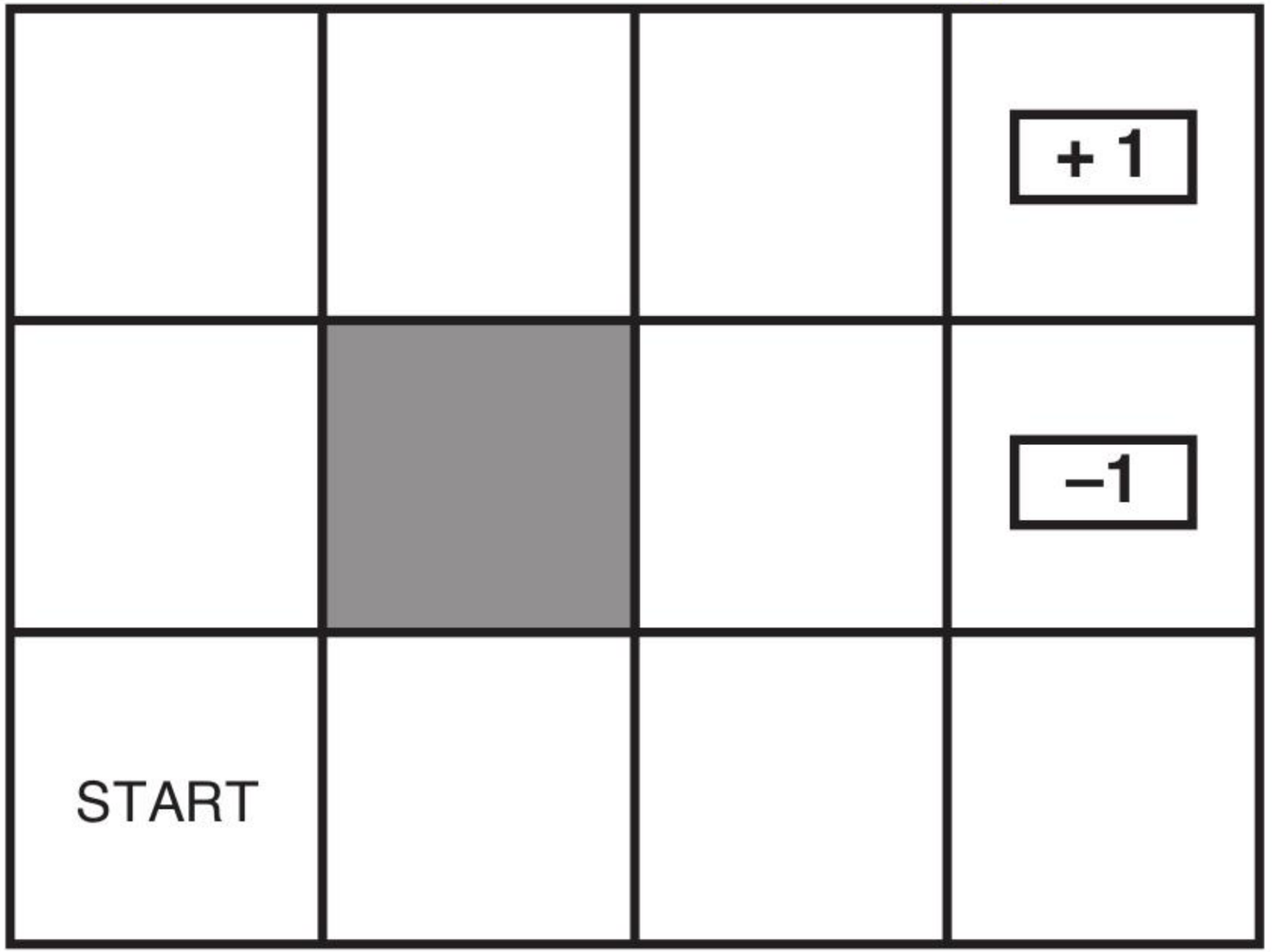

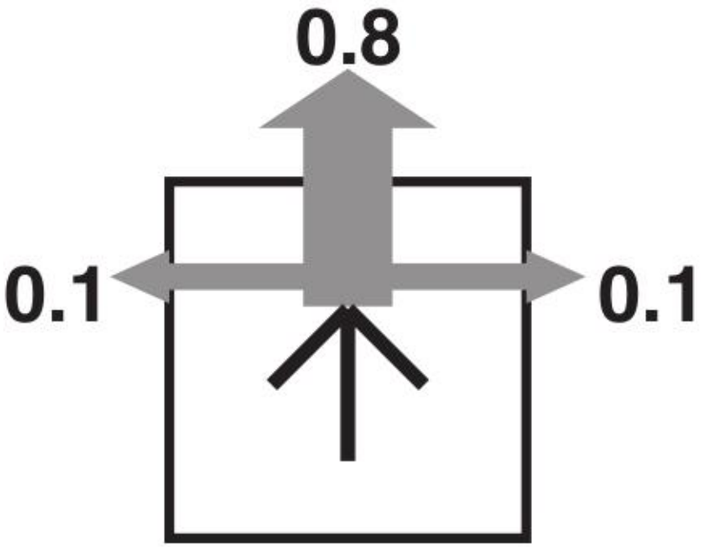

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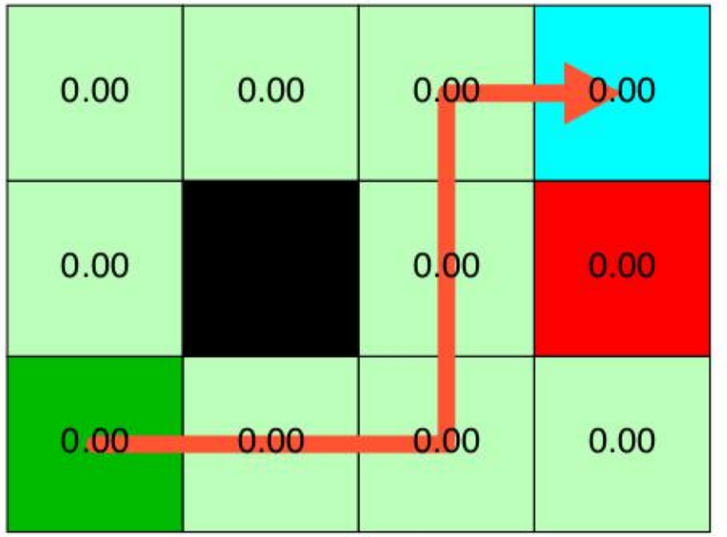
Plan? Policy

- In deterministic world: Plan – sequence of actions from Start to Goal.
- MDPs, we need a policy $\pi : S \to \mathcal{A}$.
- An action for each possible state. Why *each*?
- What is the *best* policy?



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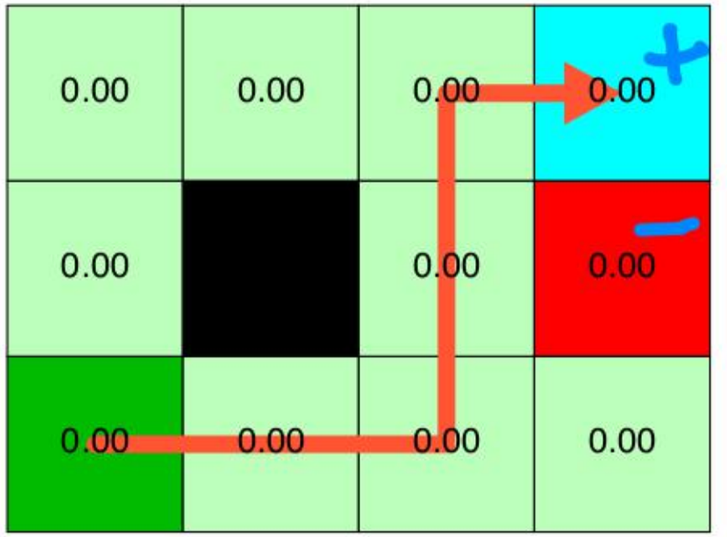

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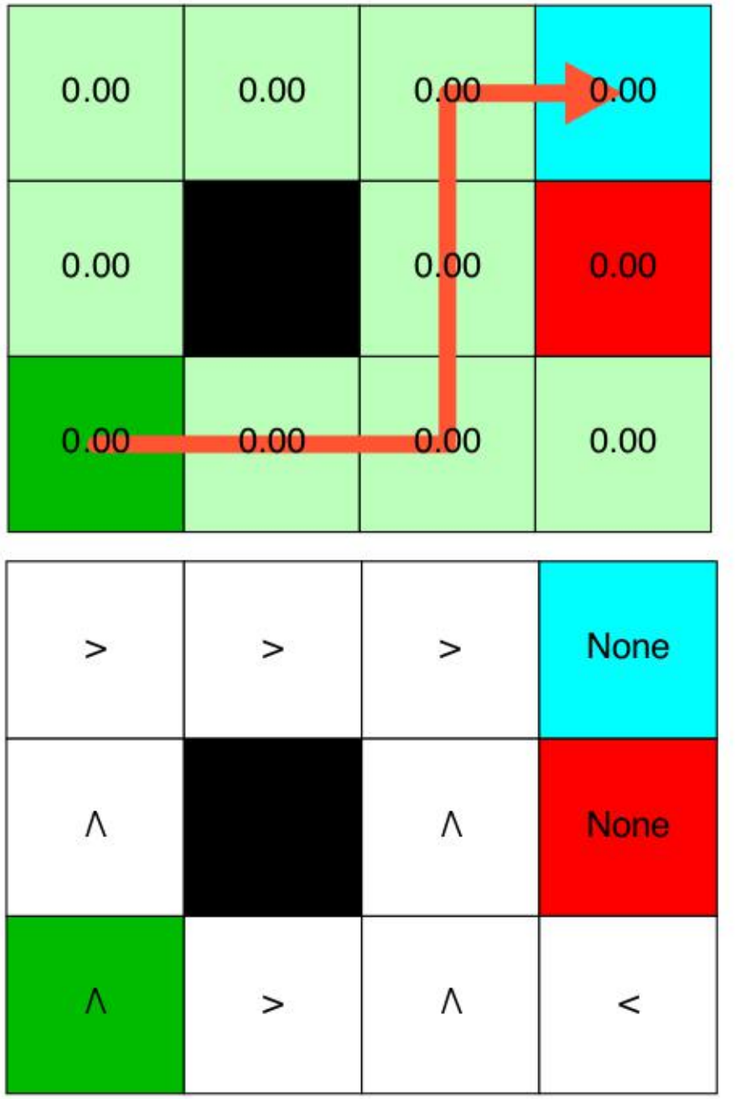

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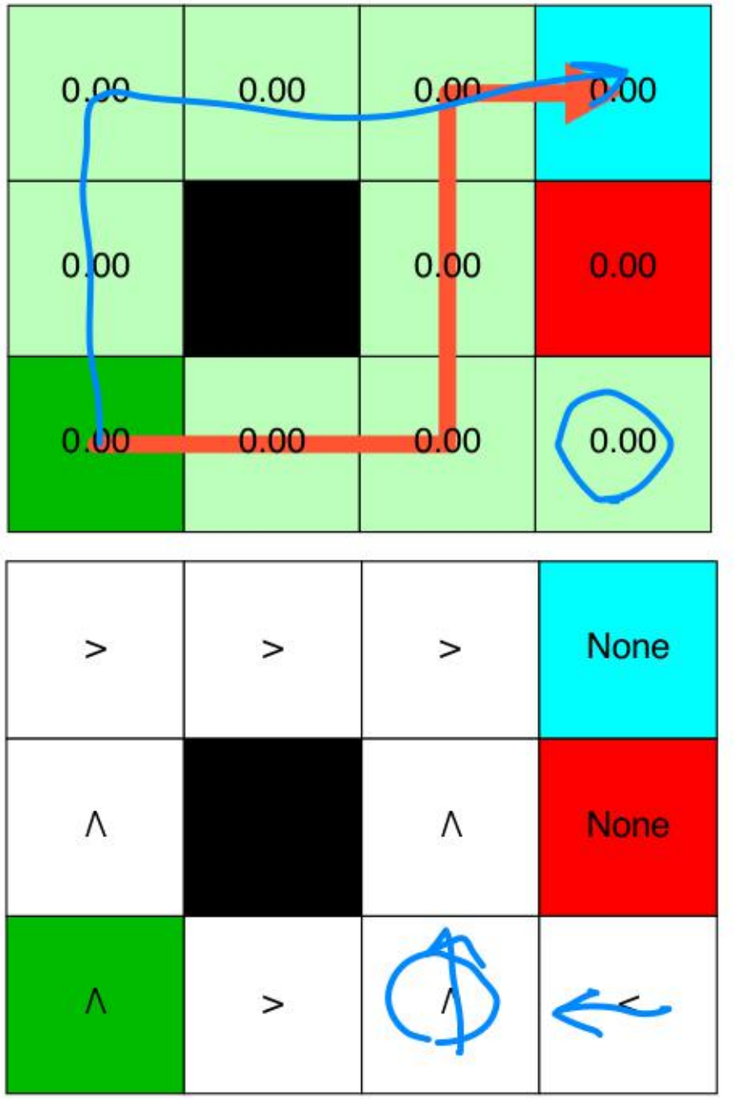

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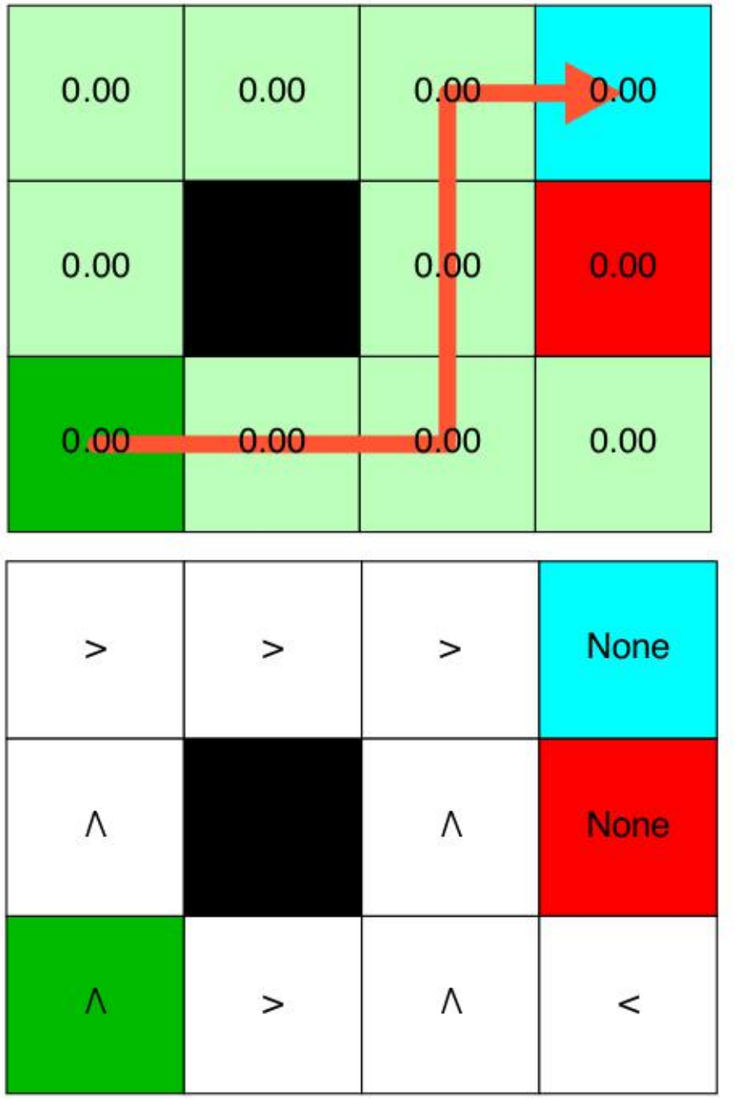

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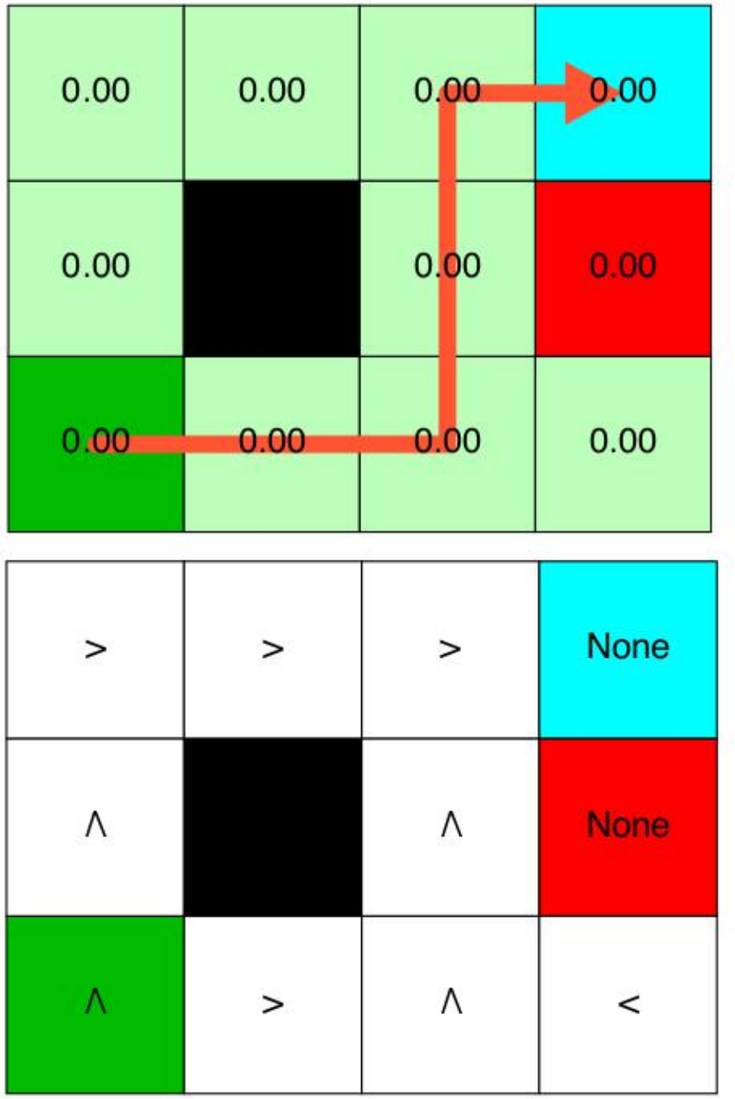
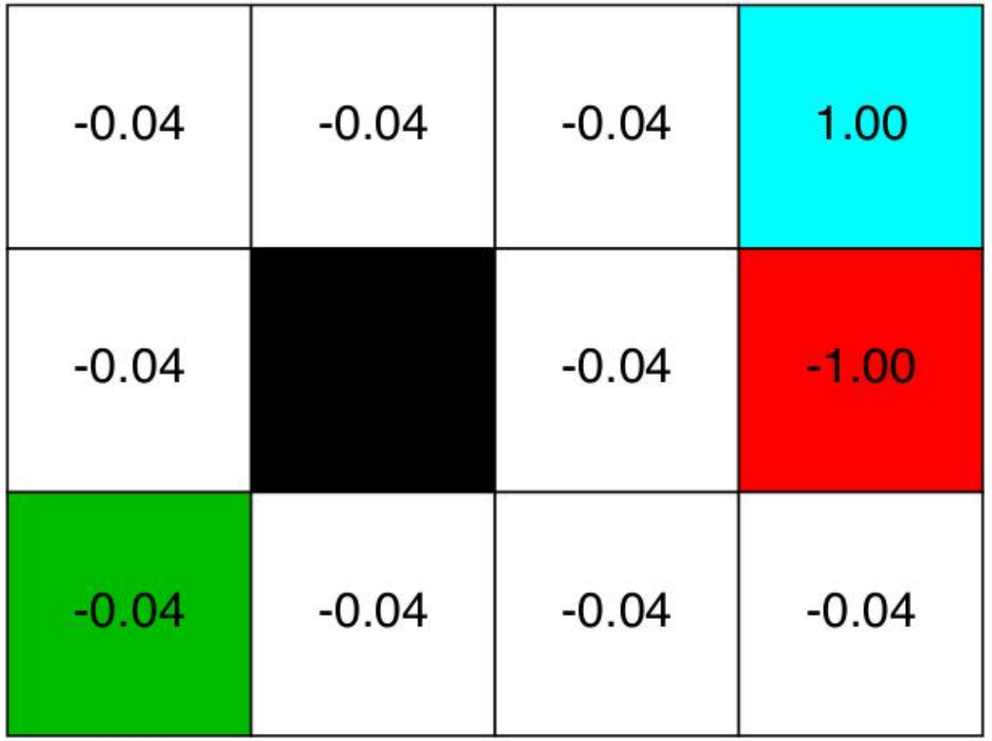


Rewards

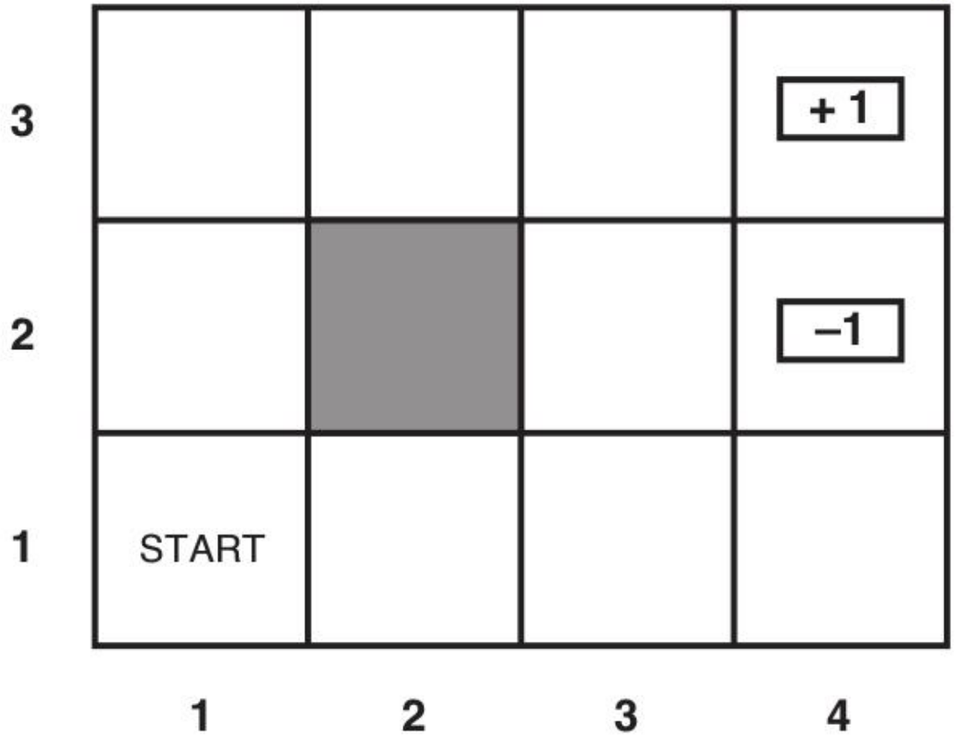


Reward : Robot/Agent takes an action a and it is **immediately** rewarded.

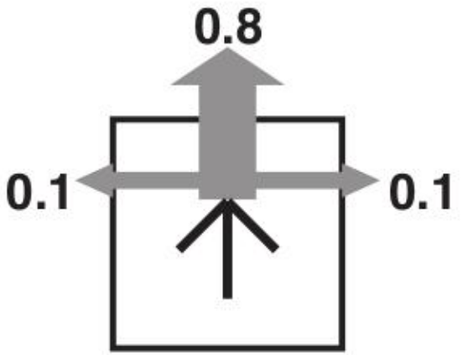
Reward function $r(s)$ (or $r(s,a)$, $r(s,a,s')$)

$$= \begin{cases} -0.04 & \text{(small penalty) for nonterminal states} \\ \pm 1 & \text{for terminal states} \end{cases}$$

Markov Decision Processes (MDPs)



(a)



(b)

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Markovian property

- Given the present state, the future and the past are independent.
- MDP: Markov means action depends only on the current state.
- In search: successor function (transition model) depends on the current state only.

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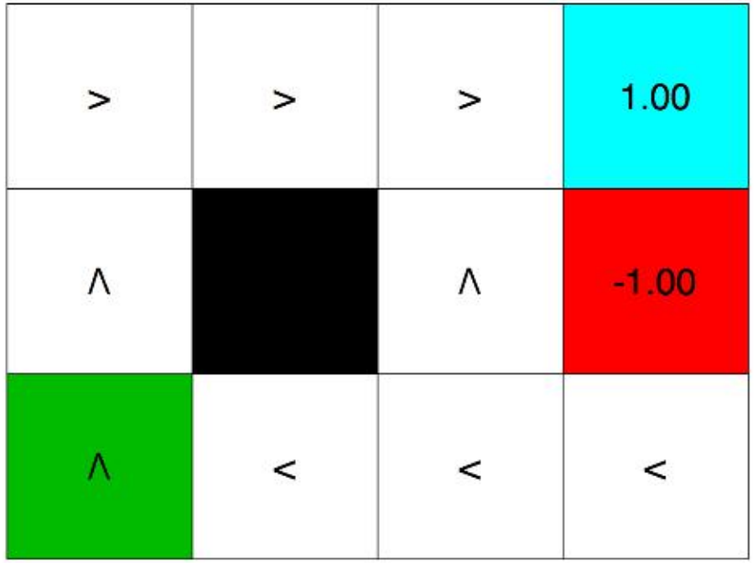
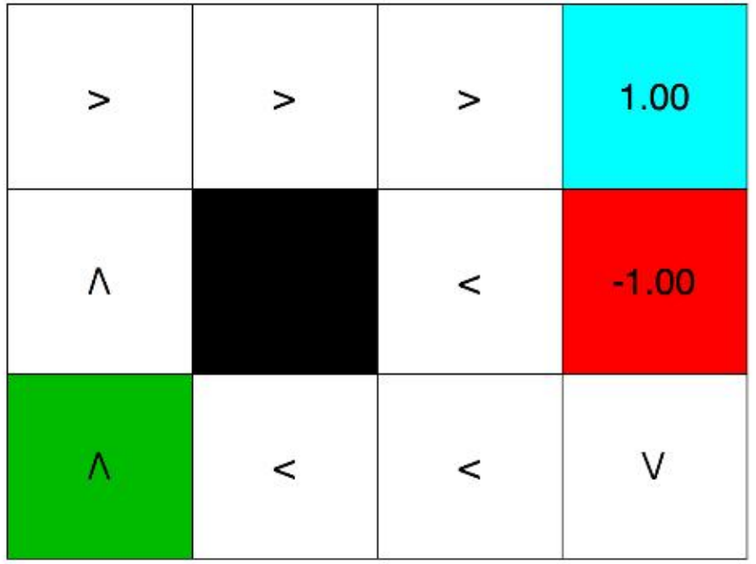
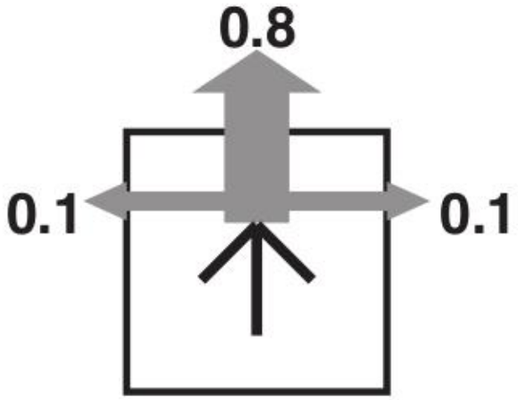
		
A	B	C
$r(s) = \{-2, 1, -1\}$ a	$r(s) = \{-0.04, 1, -1\}$ b	$r(s) = \{-0.01, 1, -1\}$ c

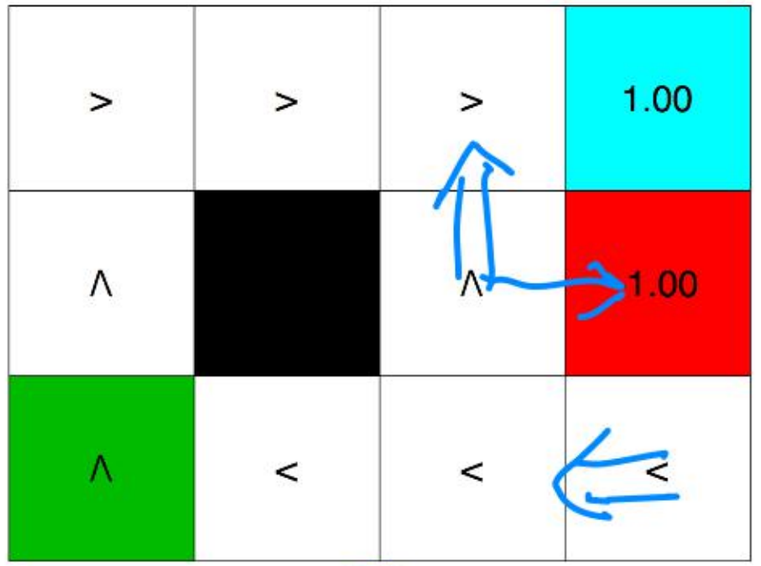
A: A-a, B-b, C-c

B: A-b, B-a, C-c

C: A-b, B-c, C-a

D: A-c, B-a, C-b





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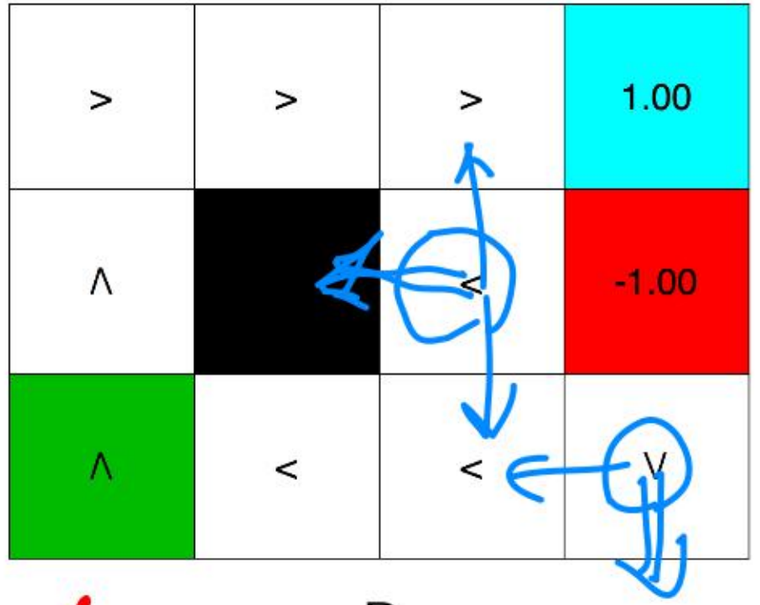

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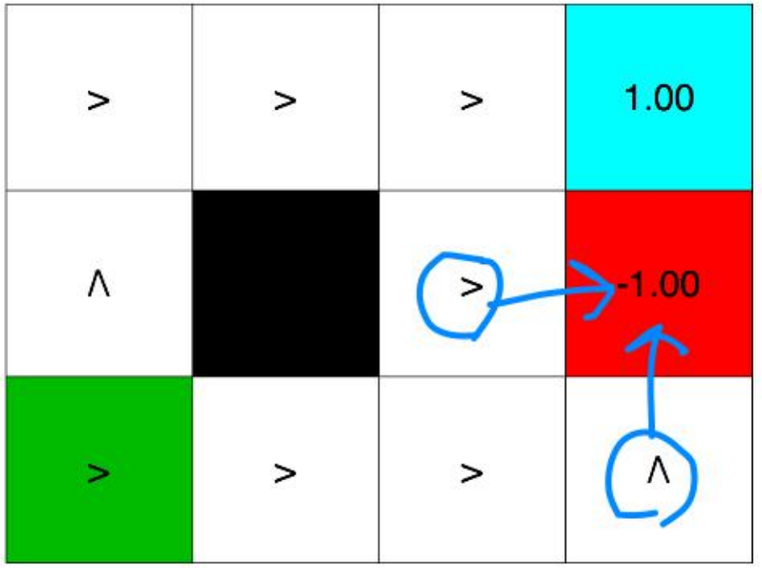

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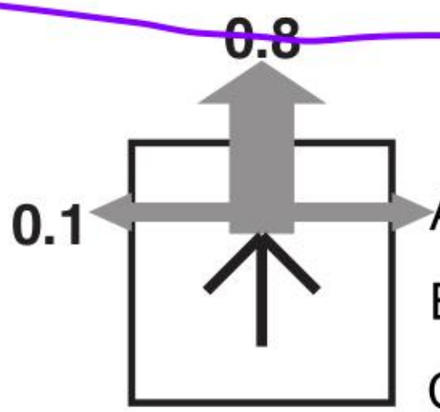

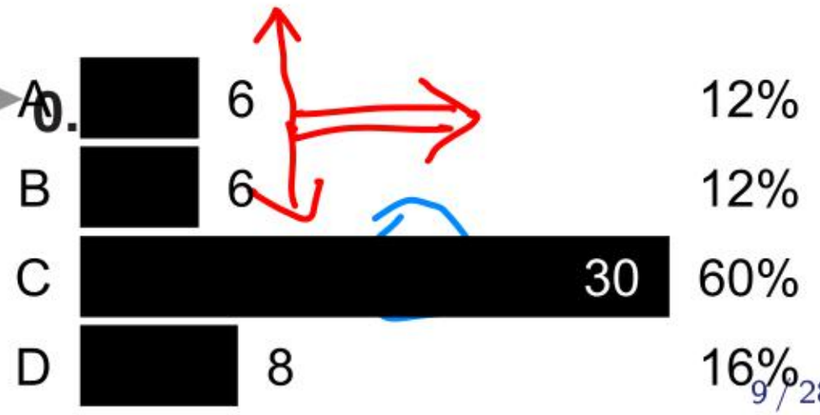

Utilities of sequences

- State reward at time/step t, R_t.
- State at time t, S_t. State sequence $[S_0, S_1, S_2, \ldots,]$

Typically, consider stationary preferences on reward sequences:

$$[R, R_1, R_2, R_3, \ldots] \succ [R, R'_1, R'_2, R'_3, \ldots] \Leftrightarrow [R_1, R_2, R_3, \ldots] \succ [R'_1, R'_2, R'_3, \ldots]$$

If stationary preferences :
Utility (h-history)
$U_h([S_0, S_1, S_2, \ldots,]) = R_1 + R_2 + R_3 + \cdots$

If the horizon is finite - limited number of steps - preferences are nonstationary (depends on how many steps left).

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Returns and Episodes

- Executing policy - sequence of states and **rewards**.
- Episode starts at t, ends at T (ending in a terminal state).
- Return (Utility) of the episode (policy execution)

$$G_t = R_{t+1} + R_{t+2} + R_{t+3} + \cdots + R_T$$

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Comparing policies; Finite vs infinite horizon

Problem: Infinite lifetime $\Rightarrow$ additive utilities are infinite.

- Finite horizon: termination at a fixed time $\Rightarrow$ nonstationary policy, $\pi(s)$ depends on the time left.
- Discounted return , $\gamma < 1, R_t \leq R_{\max}$

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- Absorbing (terminal) state.

Returns are successive steps related to each other

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MDPs recap

Markov decition processes (MDPs):

- Set of states $\mathcal{S}$
- Set of actions $\mathcal{A}$
- Transitions $p(s'|s,a)$ or $T(s,a,s')$
- Reward function $r(s,a,s')$; and discount γ

MDP quantities:

- (deterministic) Policy $\pi(s)$ – choice of action for each state
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Value functions

- Executing policy $\pi \rightarrow$ sequence of states (and rewards).
- Utility of a state sequence.
- But actions are unreliable - environment is stochastic.
- Expected return of a policy π.

Starting at time t, i.e. S_t,

$$U^{\pi}(S_t) = \mathrm{E}^{\pi}\left[\sum_{k=0}^{\infty} \gamma^k R_{t+k+1}\right]$$

Value function

$$v^{\pi}(s) = \mathrm{E}^{\pi}\left[G_t \mid S_t = s\right] = \mathrm{E}^{\pi}\left[\sum_{k=0}^{\infty} \gamma^k R_{t+k+1} \,\middle|\, S_t = s\right]$$

Action-value function (q-function)

$$q^{\pi}(s, a) = \mathrm{E}^{\pi}\left[G_t \mid S_t = s, A_t = a\right] = \mathrm{E}^{\pi}\left[\sum_{k=0}^{\infty} \gamma^k R_{t+k+1} \,\middle|\, S_t = s, A_t = a\right]$$

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$$U^{\pi}(S_t) = \mathrm{E}^{\pi}\left[\sum_{k=0}^{\infty} \gamma^k R_{t+k+1}\right]$$

Value function

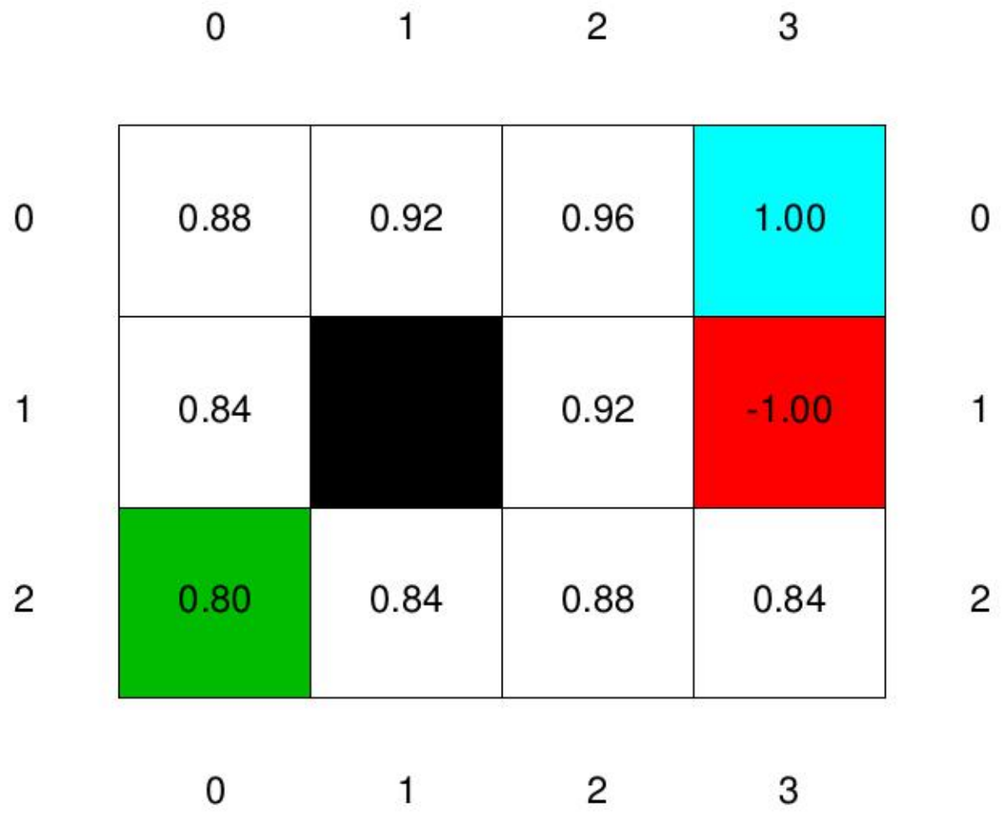
$$v^{\pi}(s) = \mathrm{E}^{\pi}\left[G_t \mid S_t = s\right] = \mathrm{E}^{\pi}\left[\sum_{k=0}^{\infty} \gamma^k R_{t+k+1} \,\middle|\, S_t = s\right]$$

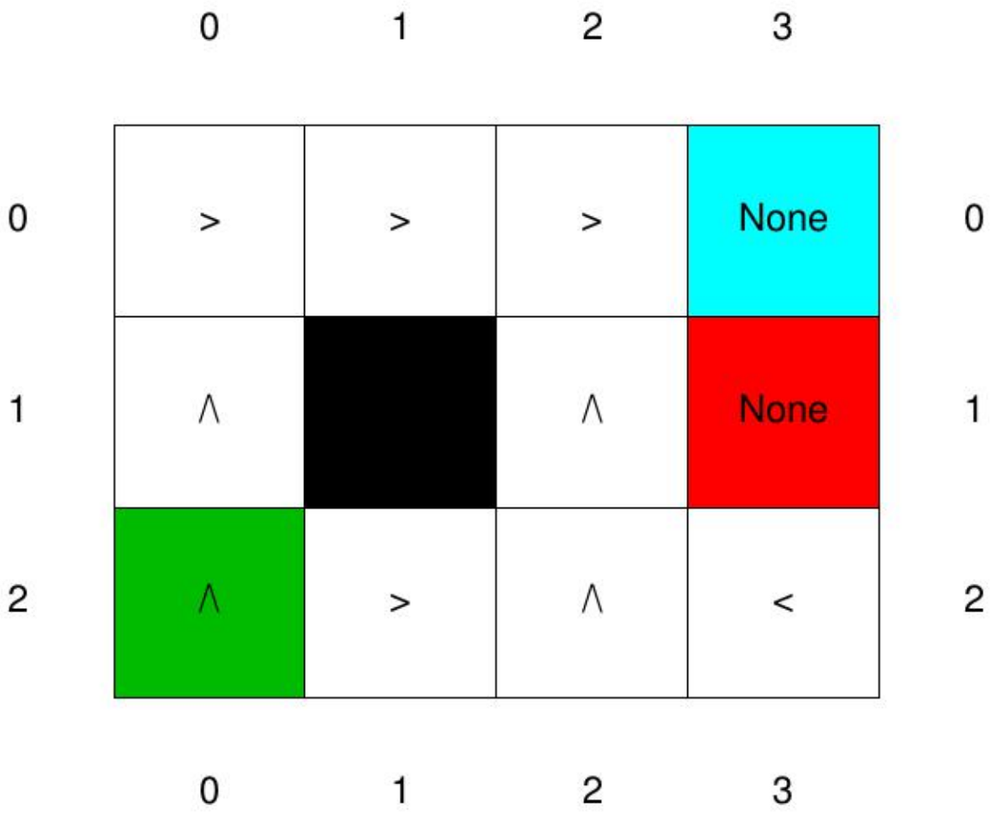
Action-value function (q-function)

$$q^{\pi}(s,a) = \mathrm{E}^{\pi}\left[G_t \mid S_t = s, A_t = a\right] = \mathrm{E}^{\pi}\left[\sum_{k=0}^{\infty} \gamma^k R_{t+k+1} \,\middle|\, S_t = s, A_t = a\right]$$

Optimal policy π^*, and optimal value $v^*(s)$

$v^*(s)$ = expected (discounted) sum of rewards (until termination) assuming *optimal* actions.





$r(s) = \{-0.04, 1, -1\}$, $\gamma = 0.999999$, $\epsilon = 0.03$, Robot *deterministic*

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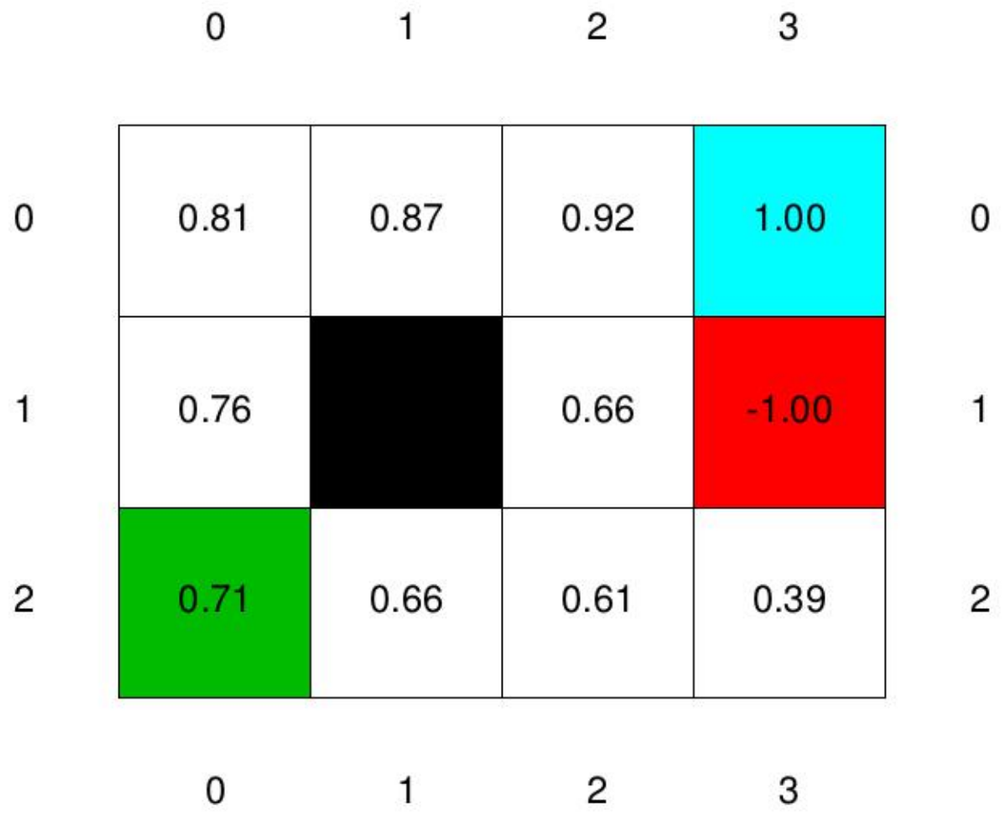

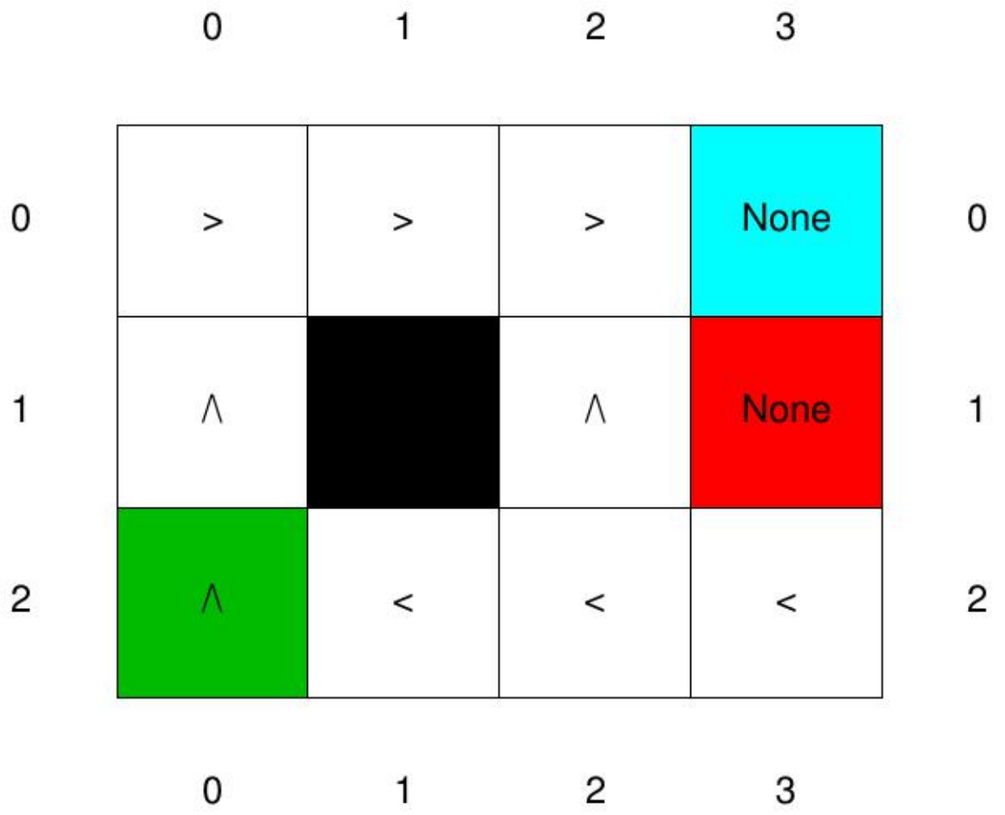

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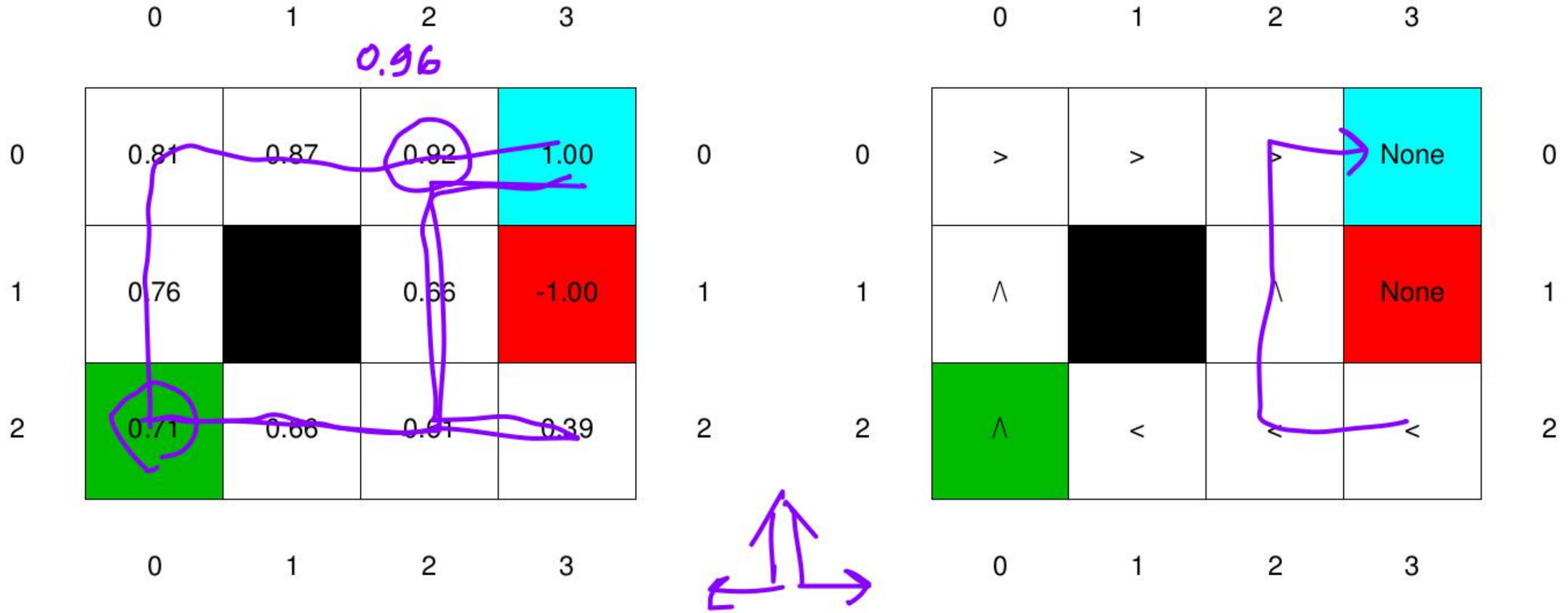

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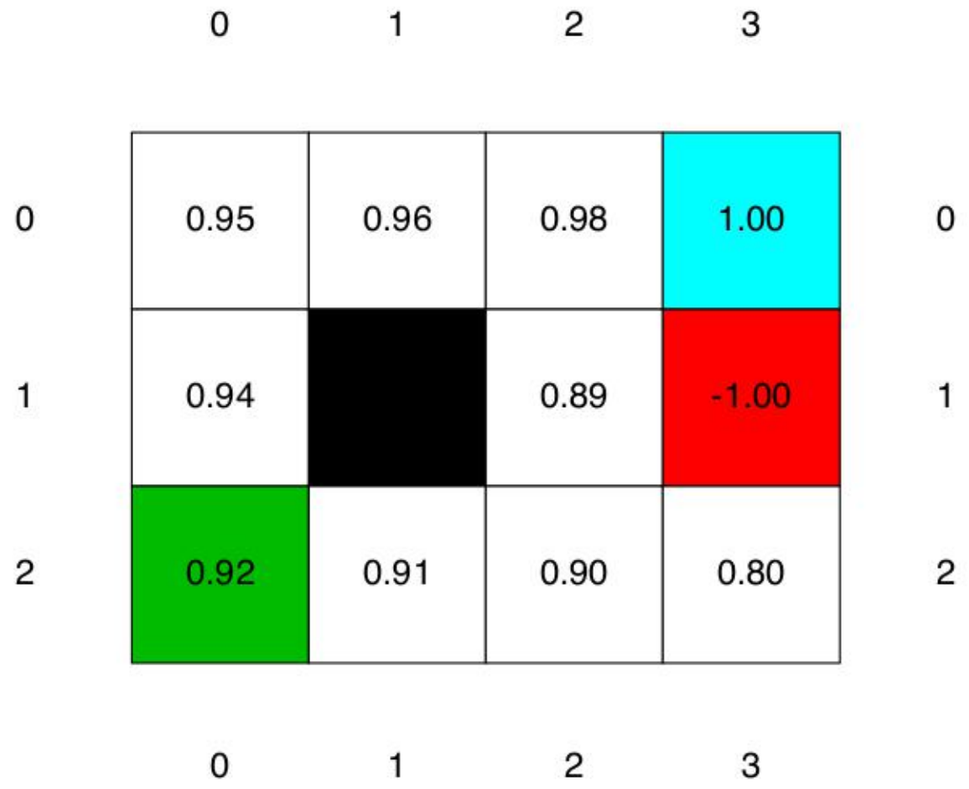
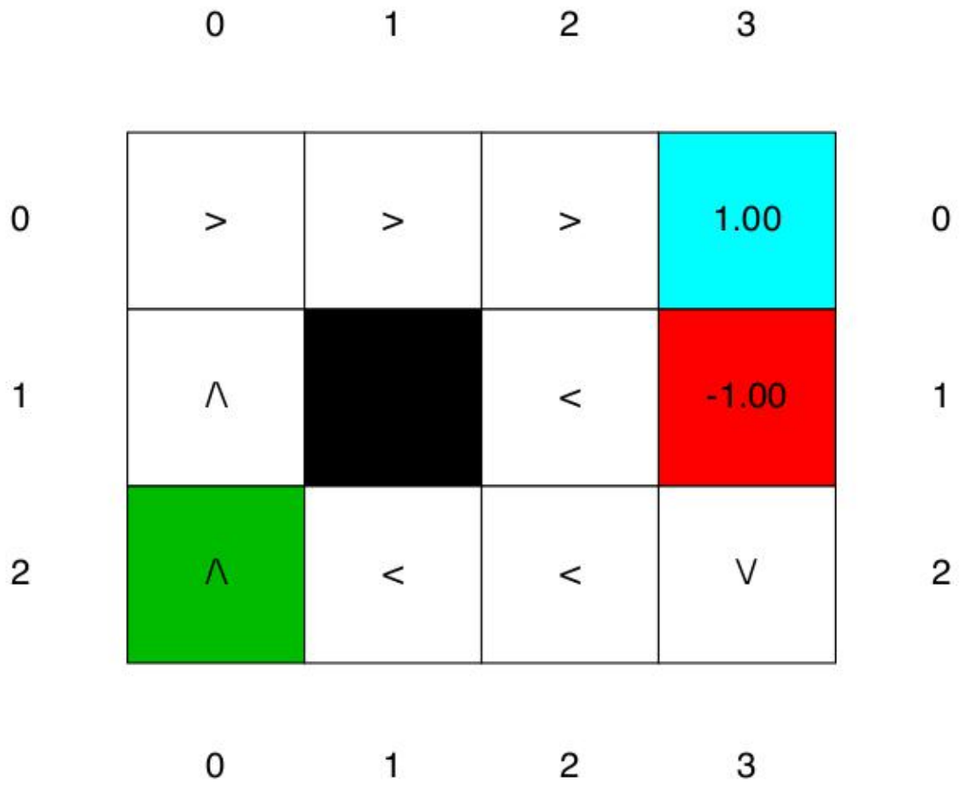
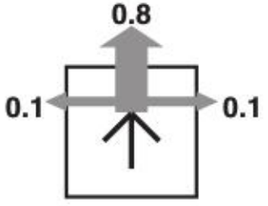




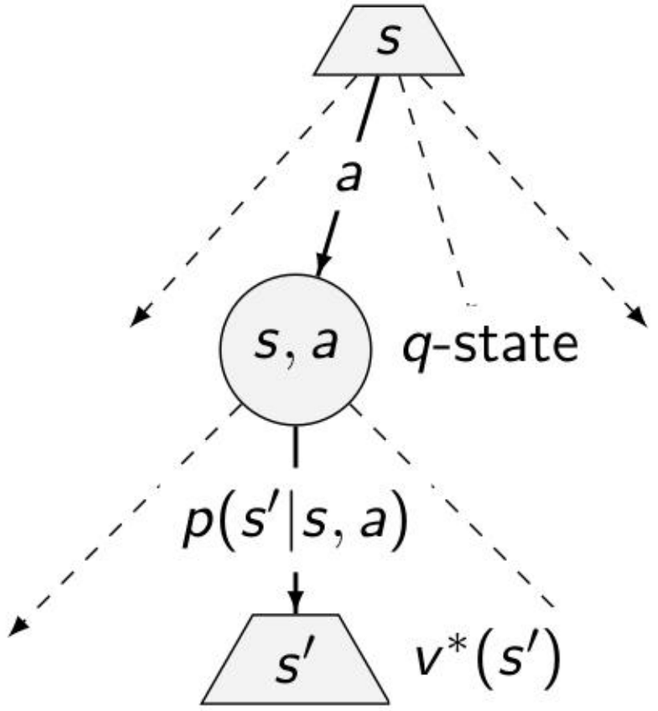
$$r(s) = \{-0.01, 1, -1\},\ \gamma = 0.999999,\ \epsilon = 0.03,$$



MDP search tree

The value of a q-state (s, a):

$$q^*(s, a) = \sum_{s'} p(s'|a, s) \left[r(s, a, s') + \gamma \, v^*(s') \right]$$



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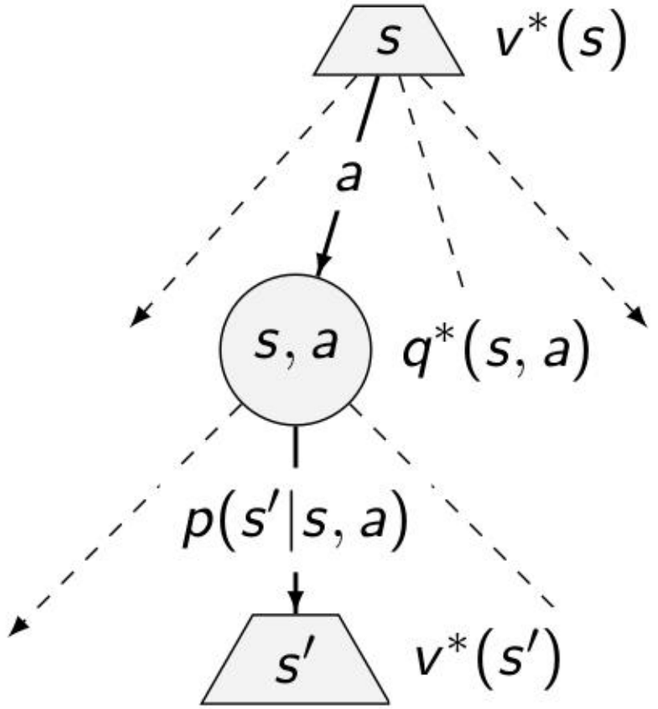
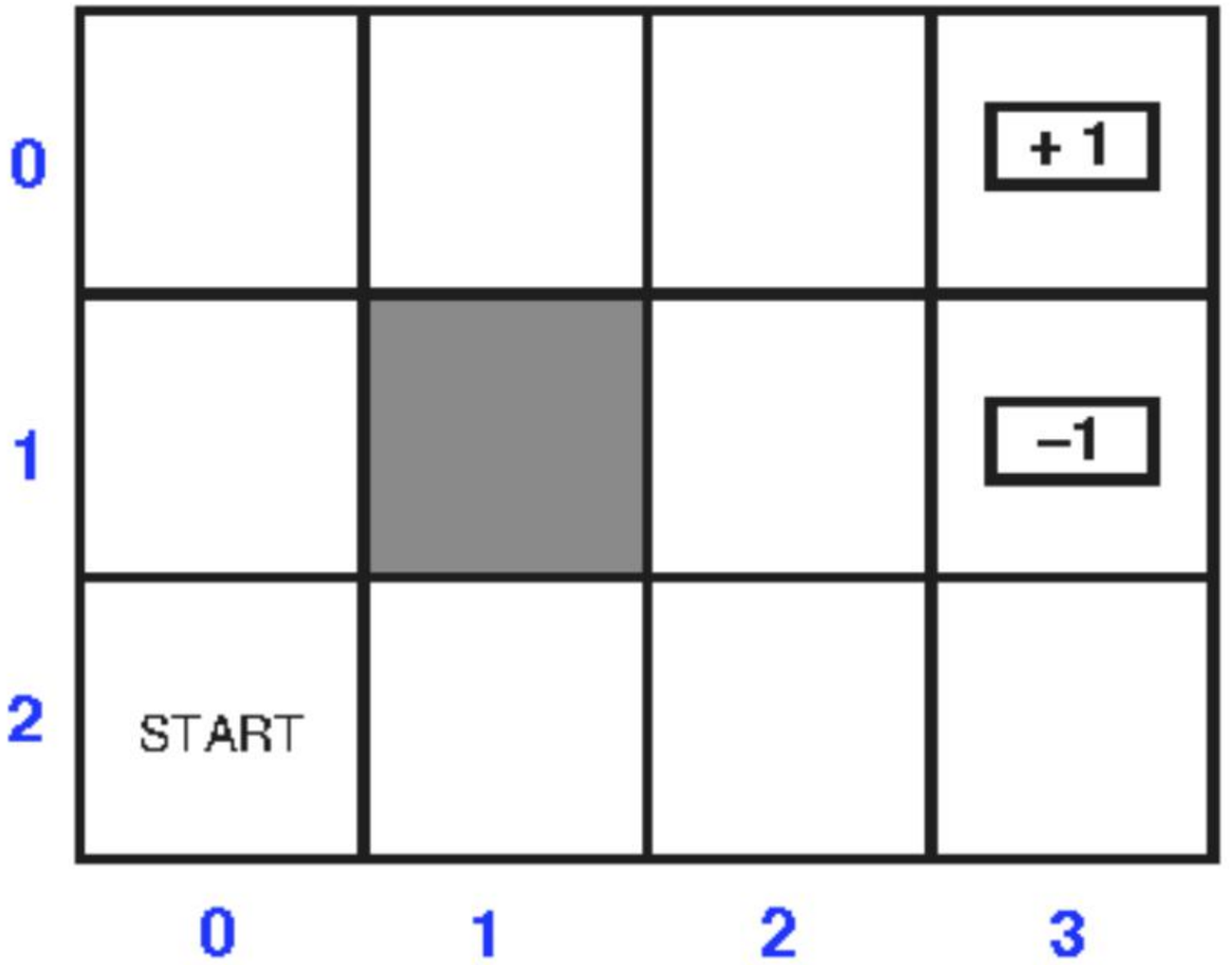
The value of a state s:

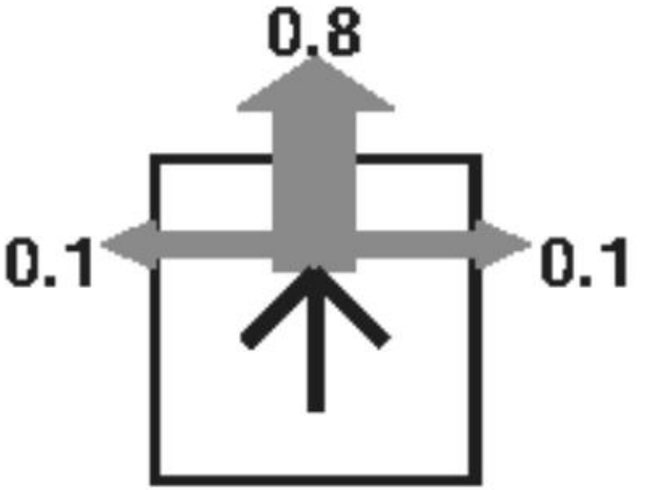
$$v^*(s) = \max_a q^*(s, a)$$



Bellman (optimality) equation

$$v^*(s) = \max_{a \in A(s)} \sum_{s'} p(s'|a,s) \left[r(s,a,s') + \gamma v^*(s') \right]$$





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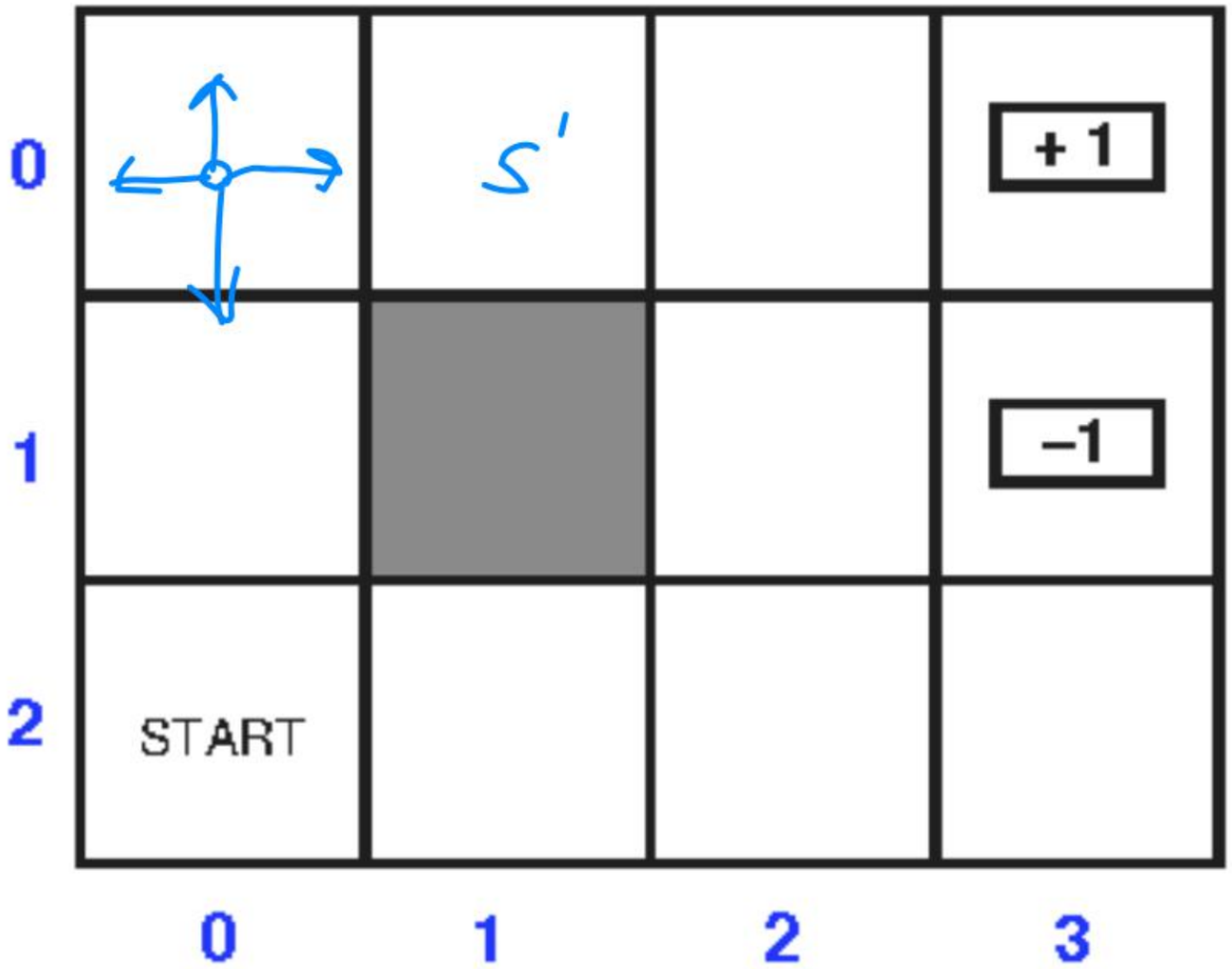

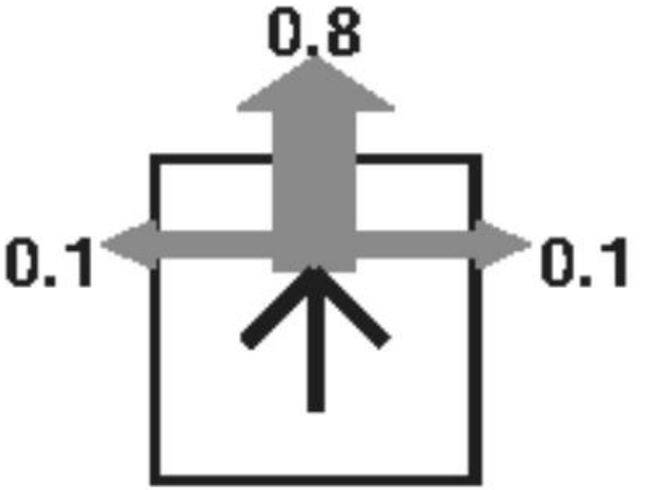

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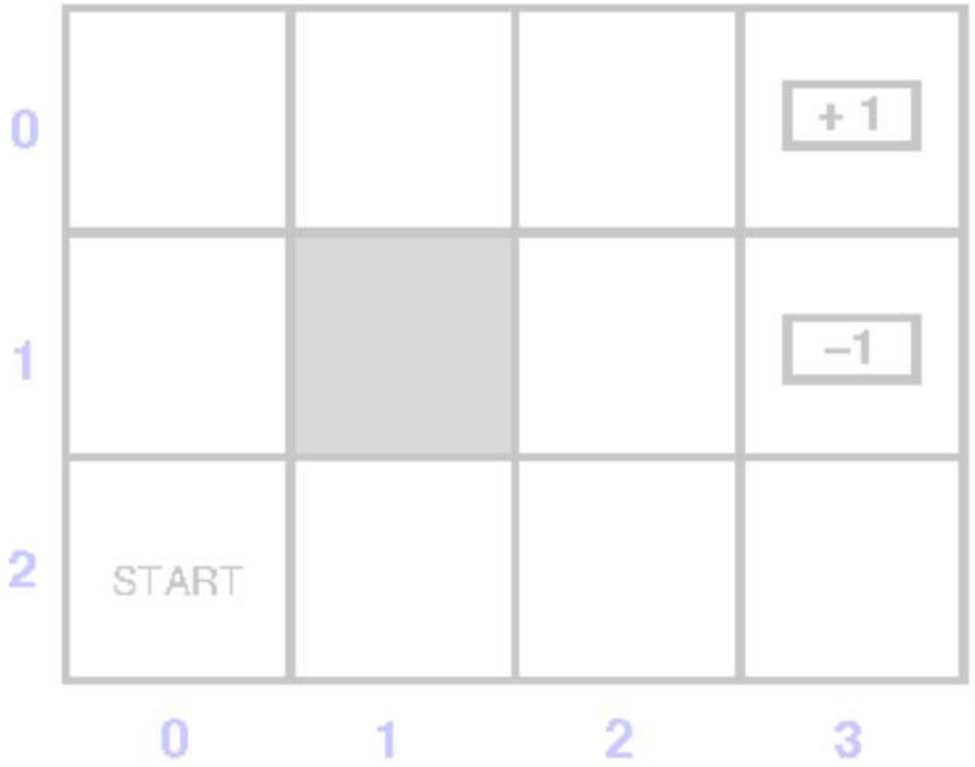

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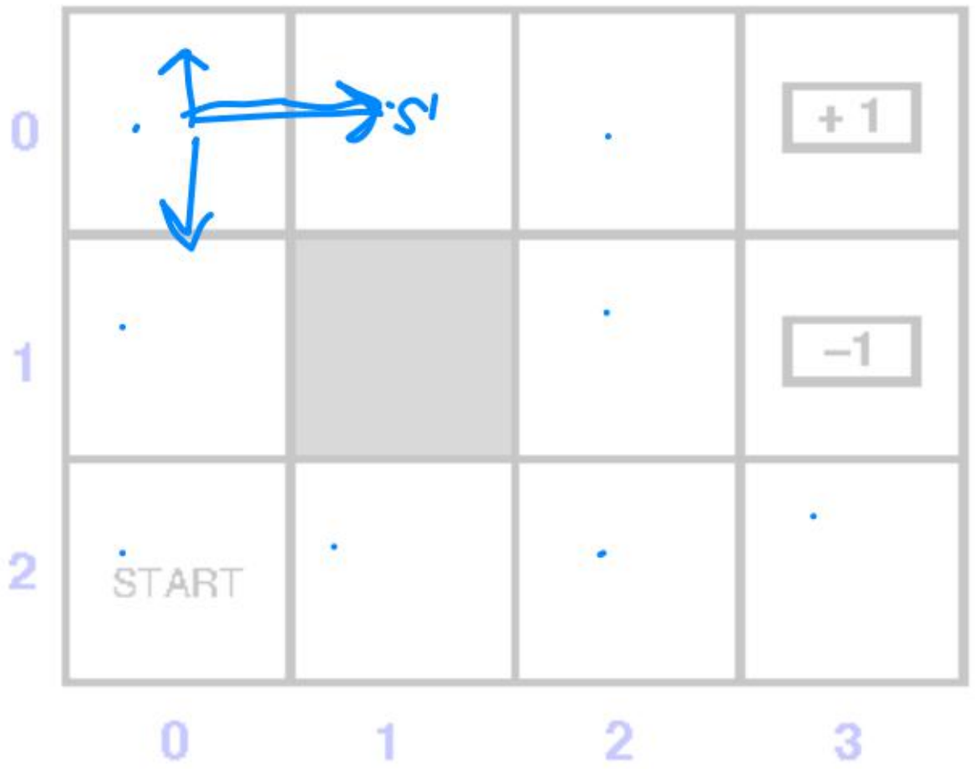

$V(0,0) =$ $a = E$

$s' 0,1$: $0.8 \left[-0.04 + \gamma V(0,1) \right]$

$s' 0,0$: $0.1 \left[-0.04 + \gamma V(0,0) \right]$

$\gamma : 9$

Value iteration

- Start with arbitrary $V_0(s)$ (except for terminals)
- Compute Bellman update (one ply of expectimax from each state)

$$V_{k+1}(s) \leftarrow R(s) + \gamma \max_{a \in A(s)} \sum_{s'} p(s'|s,a) V_k(s')$$

- Repeat until convergence

The idea: Bellman update makes local consistency with the Bellmann equation. Everywhere locally consistent $\Rightarrow$ globally optimal.

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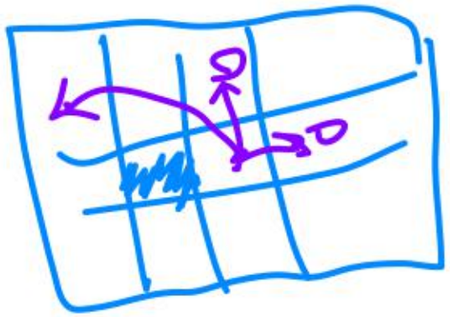
Value iteration - Complexity of one estimation sweep

$$V_{k+1}(s) \leftarrow R(s) + \gamma \max_{a \in A(s)} \sum_{s'} p(s'|s,a) V_k(s')$$

- $O(AS)$
- $O(S^2)$
- $O(AS^2)$
- $O(A^2S^2)$

Value iteration - Complexity of one estimation sweep



$$V_{k+1}(s) \leftarrow R(s) + \gamma \max_{a \in A(s)} \sum_{s'} p(s'|s,a) V_k(s')$$

- A ▶ $O(AS)$
- B ▶ ~~$O(S^2)$~~
- C ▶ $O(AS^2)$
- D ▶ $O(A^2S^2)$

Convergence cont'd

$V_{k+1} \leftarrow BV_k \ldots B$ as the Bellman update $V_{k+1}(s) \leftarrow R(s) + \gamma \max_{a \in A(s)} \sum_{s'} p(s'|s,a) V_k(s')$

$\|BV_k - BV'_k\| \leq \gamma \|V_k - V'_k\|$
$\|BV_k - V_{\text{true}}\| \leq \gamma \|V_k - V_{\text{true}}\|$
Rewards are bounded, at the beginning then Value error is
$\|V_0 - V_{true}\| \leq \frac{2R_{\max}}{1-\gamma}$
We run N iterations and reduce the error by factor γ in each and want to stop the error is below ϵ:
$\gamma^N 2R_{\max}/(1-\gamma) \leq \epsilon$ Taking logs, we find: $N \geq \frac{\log(2R_{\max}/\epsilon(1-\gamma))}{\log(1/\gamma)}$
To stop the iteration we want to find a bound relating the error to the size of *one* Bellman update for any given iteration.
We stop if

$$\|V_{k+1} - V_k\| \leq \frac{\epsilon(1-\gamma)}{\gamma}$$

then also: $\|V_{k+1} - V_{\text{true}}\| \leq \epsilon$ Proof on the next slide

Convergence cont'd

$\|V_{k+1} - V_{\text{true}}\| \leq \epsilon$ is the same as $\|V_{k+1} - V_{\infty}\| \leq \epsilon$
Assume $\|V_{k+1} - V_k\| = \text{err}$
In each of the following iteration steps we reduce the error by the factor γ (because $\|BV_k - V_{\text{true}}\| \leq \gamma\|V_k - V_{\text{true}}\|$). Till ∞, the total sum of reduced errors is:

$$\text{total} = \gamma\text{err} + \gamma^2\text{err} + \gamma^3\text{err} + \gamma^4\text{err} + \cdots = \frac{\gamma\text{err}}{(1-\gamma)}$$

We want to have total $< \epsilon$.

$$\frac{\gamma\text{err}}{(1-\gamma)} < \epsilon$$

From it follows that

$$\text{err} < \frac{\epsilon(1-\gamma)}{\gamma}$$

Hence we can stop if $\|V_{k+1} - V_k\| < \epsilon(1-\gamma)/\gamma$

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Value iteration algorithm

function VALUE-ITERATION(env,ϵ) **returns:** state values V

input: env - MDP problem, ϵ

$V' \leftarrow 0$ in all states

repeat ▷ iterate values until convergence

$V \leftarrow V'$ ▷ keep the last known values

$\delta \leftarrow 0$ ▷ reset the max difference

for each state s **in** S **do**

$V'[s] \leftarrow R(s) + \gamma \max_{a \in A(s)} \sum_{s'} P(s'|s,a) V(s')$

if $|V'[s] - V[s]| > \delta$ **then** $\delta \leftarrow |V'[s] - V[s]|$

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until $\delta < \epsilon(1-\gamma)/\gamma$

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