Reinforcement learning II

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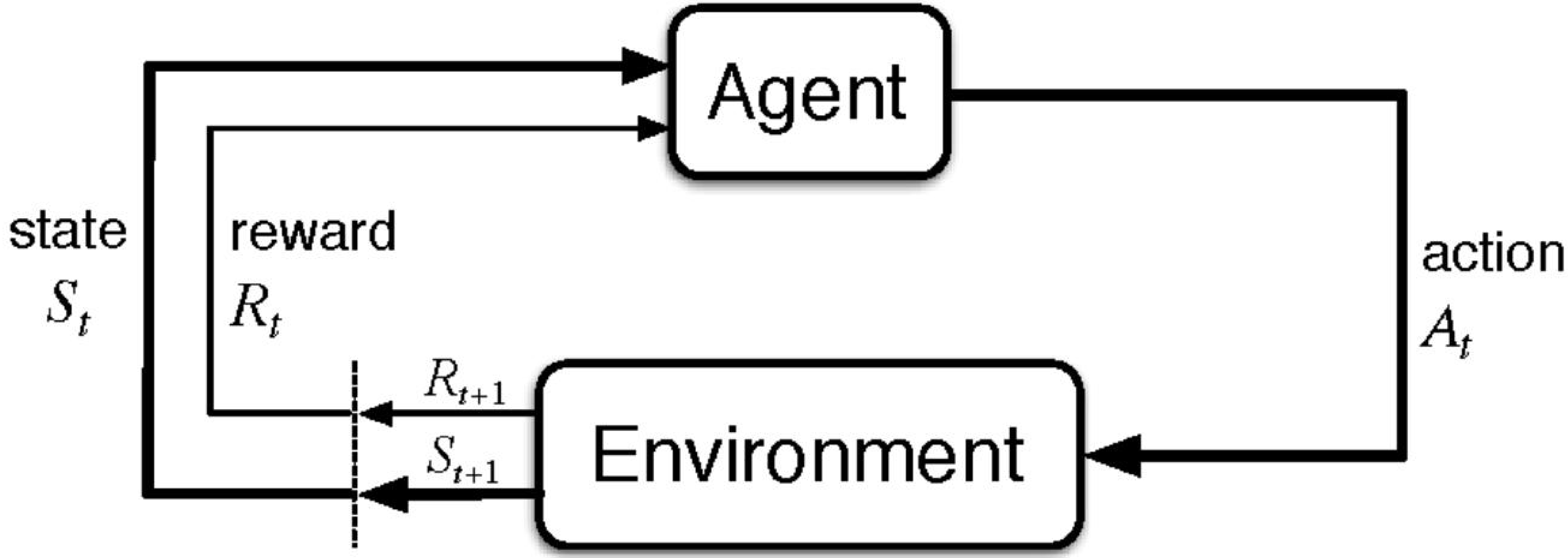
Recap: Reinforcement Learning



[1]

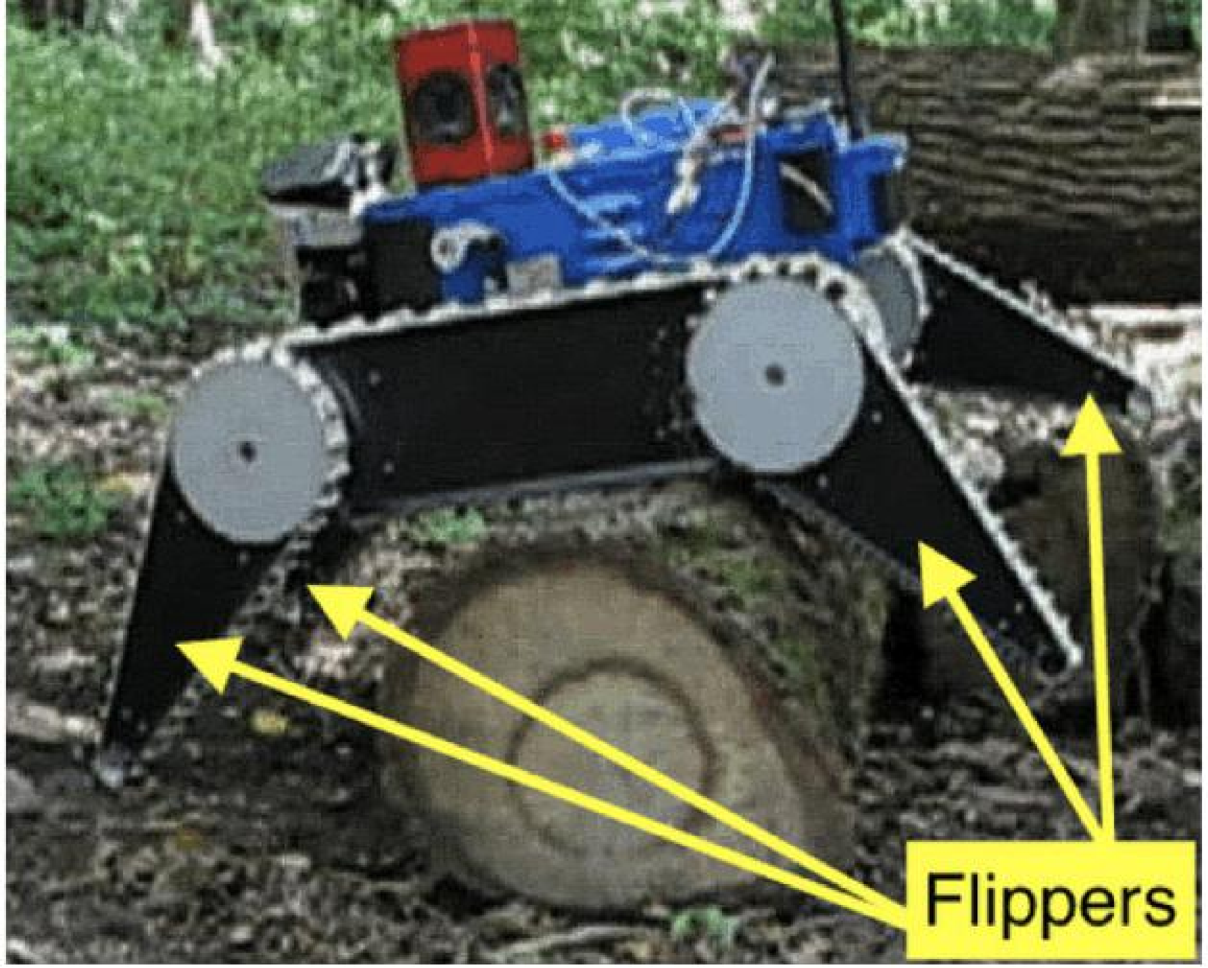
- Feedback in form of Rewards
- Learn to act so as to maximize sum of expected rewards.
- In `kuimaze` package, `env.step(action)` is the method.

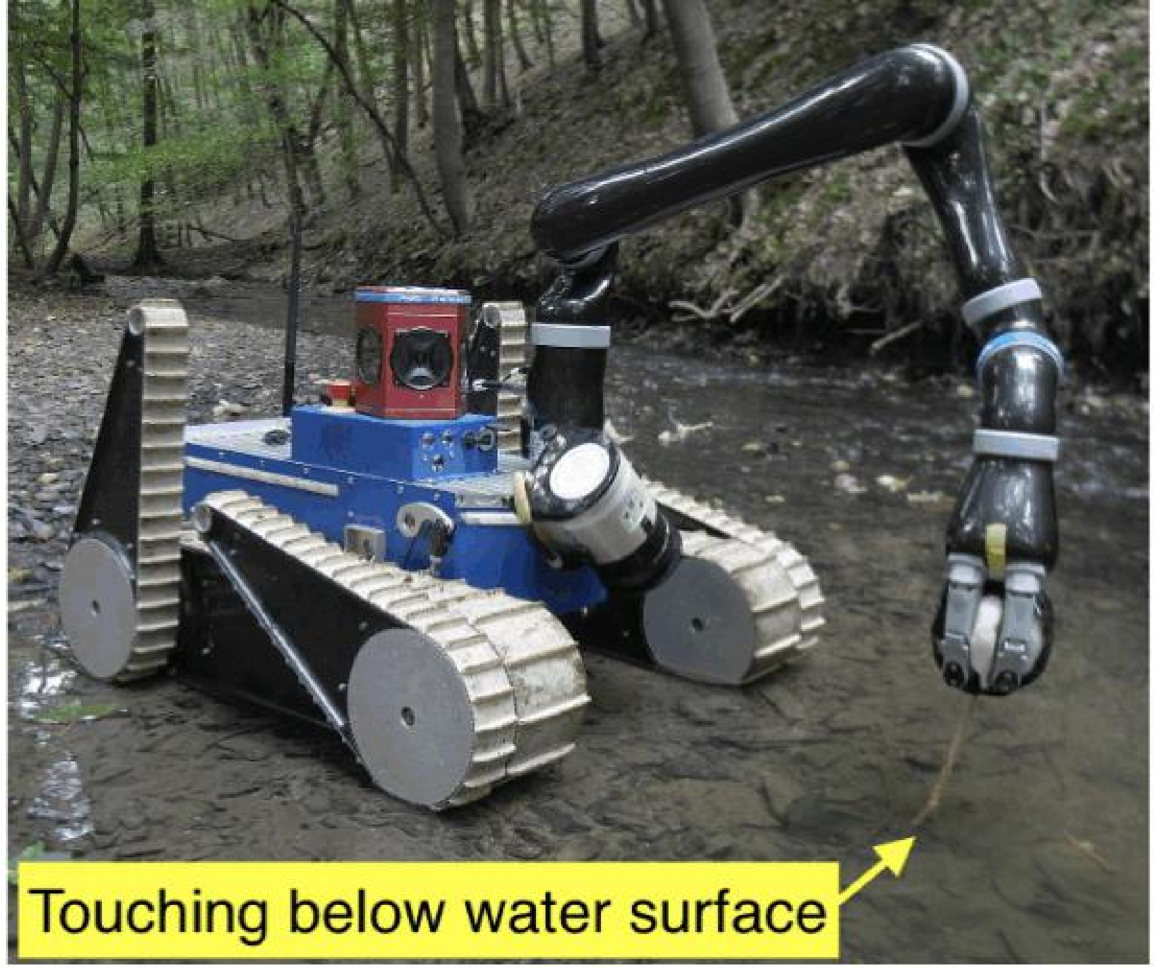
[1] Scheme from [2]

Learning to control flippers

http://cyber.felk.cvut.cz/vras/

- What are the states?
- How to design rewards?
- How to perform training episodes (roll-outs)?
- Simulator to reality gap.

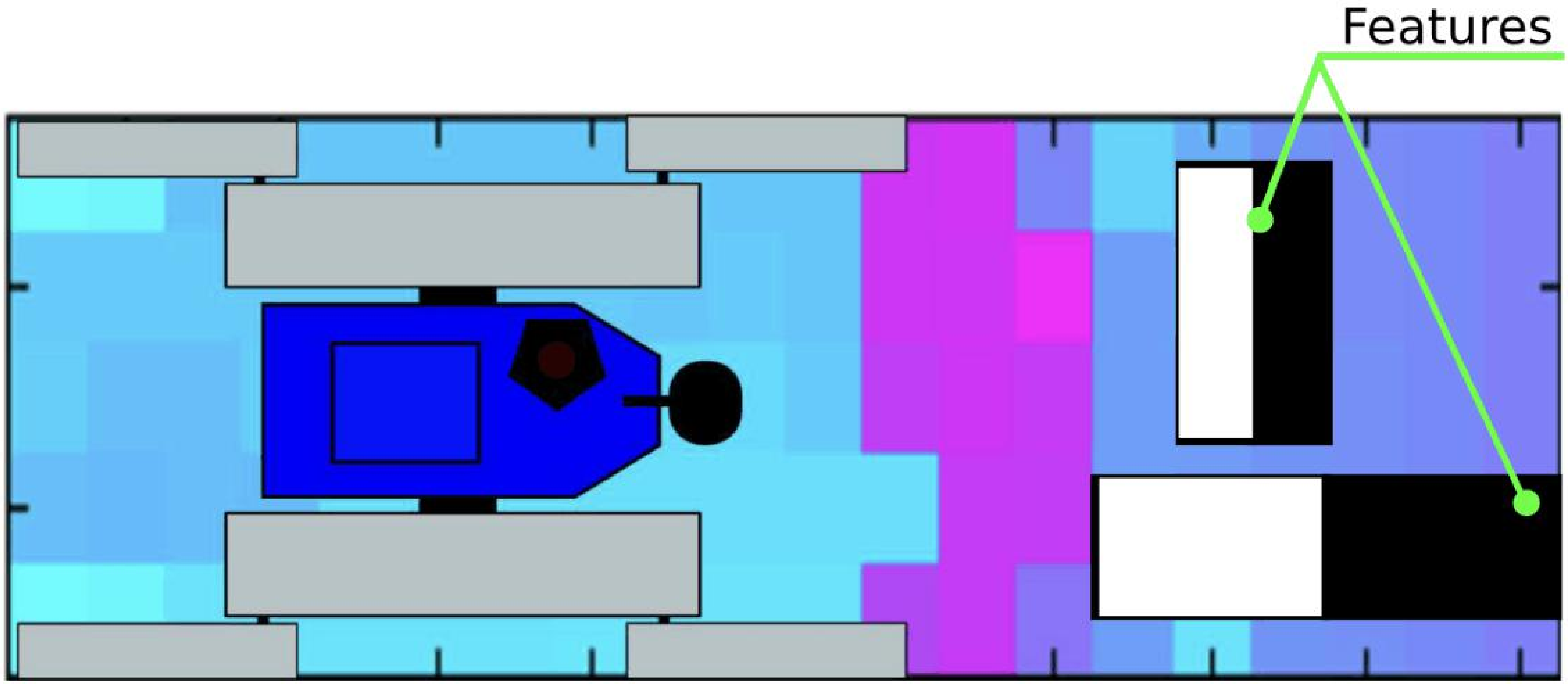




- Construction: $2\times$ main tracks, $4\times$ subtracks (flippers), differential break
  great stability and climbing capability
- Sensor suite: SICK LMS-151 range finder, Ladybug omnicam, Xsens MTi-G IMU
  3D sensing and localization
- Control inputs: Velocity vector, $4\times$**flipper angle**, $4\times$ **flipper stiffness**, differential break (0/1)
  difficult to control all of them manually!

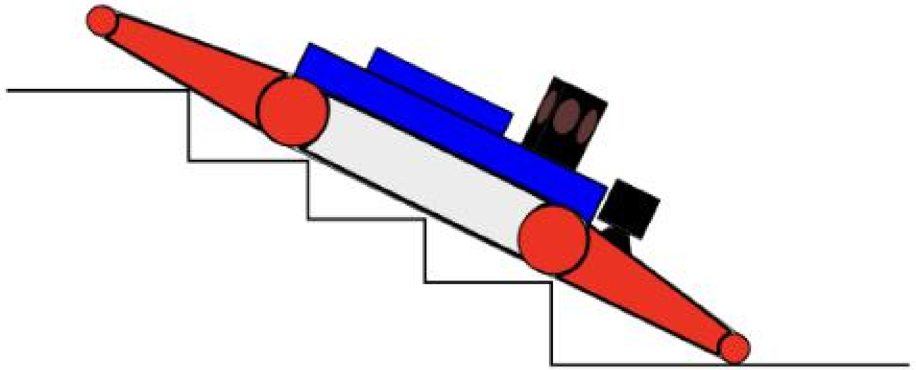
State $\mathbf{s} \in \mathcal{S} \subset \mathbb{R}^n$ concatenates:

- Proprioceptive measurements: roll, pitch, torques, velocity, acceleration.
- Local exteroceptive measurements. features on digital elevation map with fixed size.

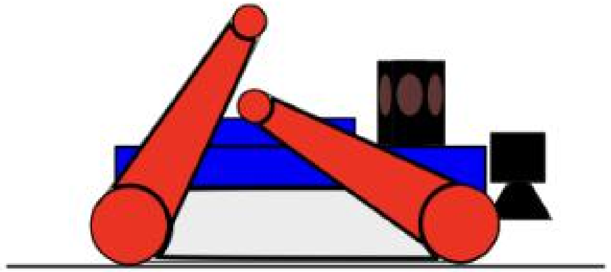


Instead of $\mathbf{a} \in \mathcal{A} \subset \mathbb{R}^8$ we consider only 5 configurations[2]:

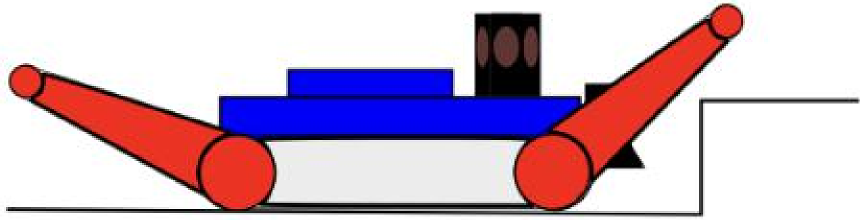
$$\mathcal{A} = \{\text{I-shape}\,, \text{V-shape}\,, \text{L-shape}\,, \text{U-shape soft}\,, \text{U-shape hard}\}$$



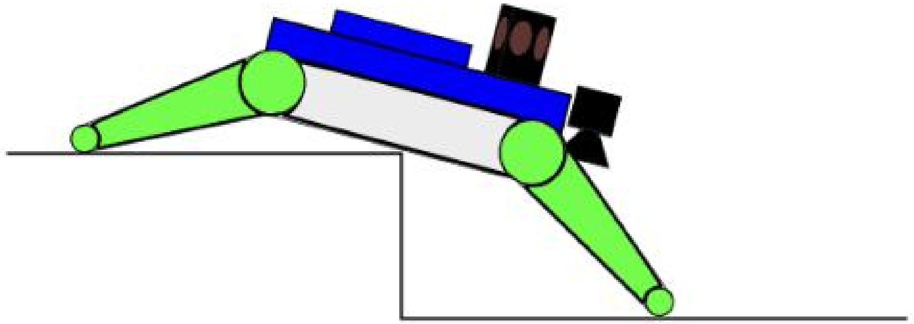
I-shape
(Maximizes traction)



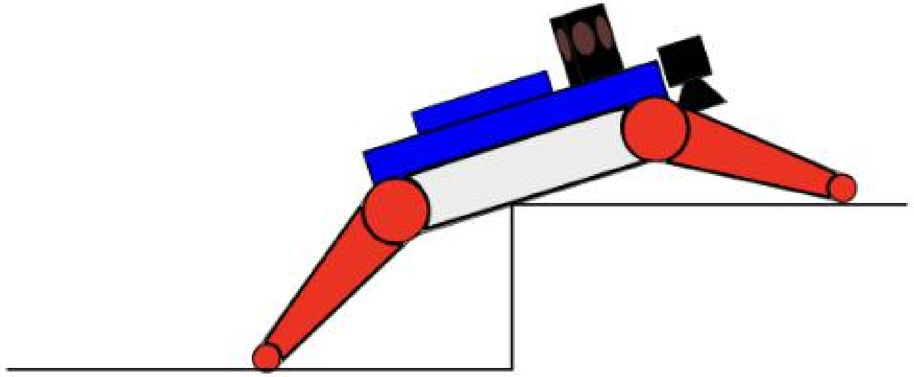
V-shape
(Provides observability)



L-shape
(Forward approach)



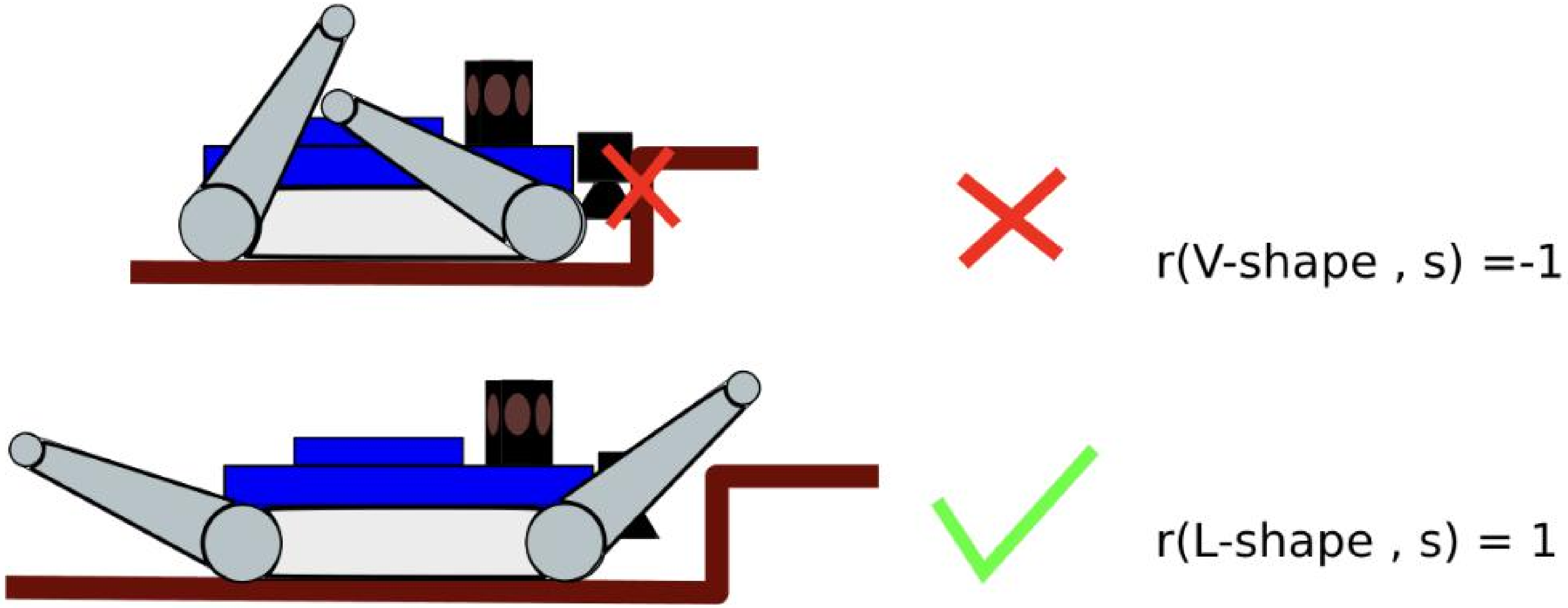
U-shape soft
(Smooth climbing down)



U-shape hard
(Lifts the body up)

Reward $r(a, \mathbf{s}) : \mathcal{A} \times \mathcal{S} \rightarrow \mathbb{R}$ is a weighted sum of following contributions:

1. Safe pitch and roll reward, avoiding tipping over
2. Smoothness reward, suppresses body hits
3. Speed reward, drives robot forward
4. User denoted reward (penalty) indicating the success (failure) of the particular maneuver indicates failure/possible damages



From off-line (MDPs) to on-line (RL)

Markov decision process – MDPs. Off-line search, we know:

- A set of states $s \in \mathcal{S}$ (map)
- A set of actions per state. $a \in \mathcal{A}$
- A transition model $p(s'|s, a)$ (robot)
- A reward function $r(s, a, s')$ (map, robot)

Looking for the optimal policy $\pi(s)$. We can plan/search before the robot enters the environment.

On-line problem:

- Transition p and reward r functions not known.
- Agent/robot must act and learn from experience.

(Transition) Model-based learning

The main idea: Do something and:

- Learn an approximate model from experiences.
- Solve as if the model were correct.

Learning MDP model:

- Try s, a, observe s', count s, a, s'.
- Normalize to get and estimate of $p(s'|s, a)$
- Discover each $r(s, a, s')$ when experienced.

Solve the learned MDP.

Model-free learning

- r, p not known.
- Move around, observe
- And learn on the way.
- **Goal:** learn the state value $v(s)$ or (better) q-value $q(s, a)$ functions.

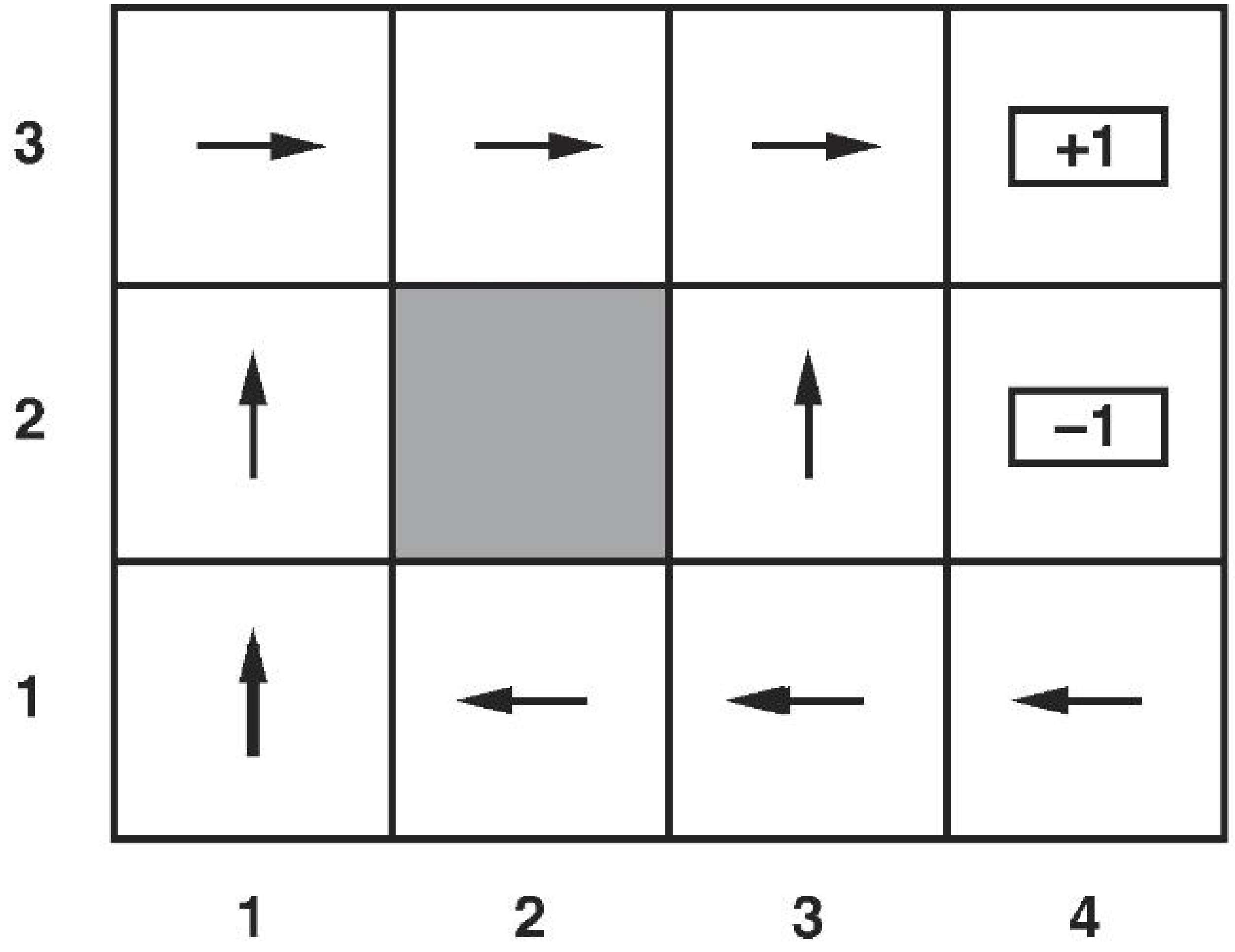
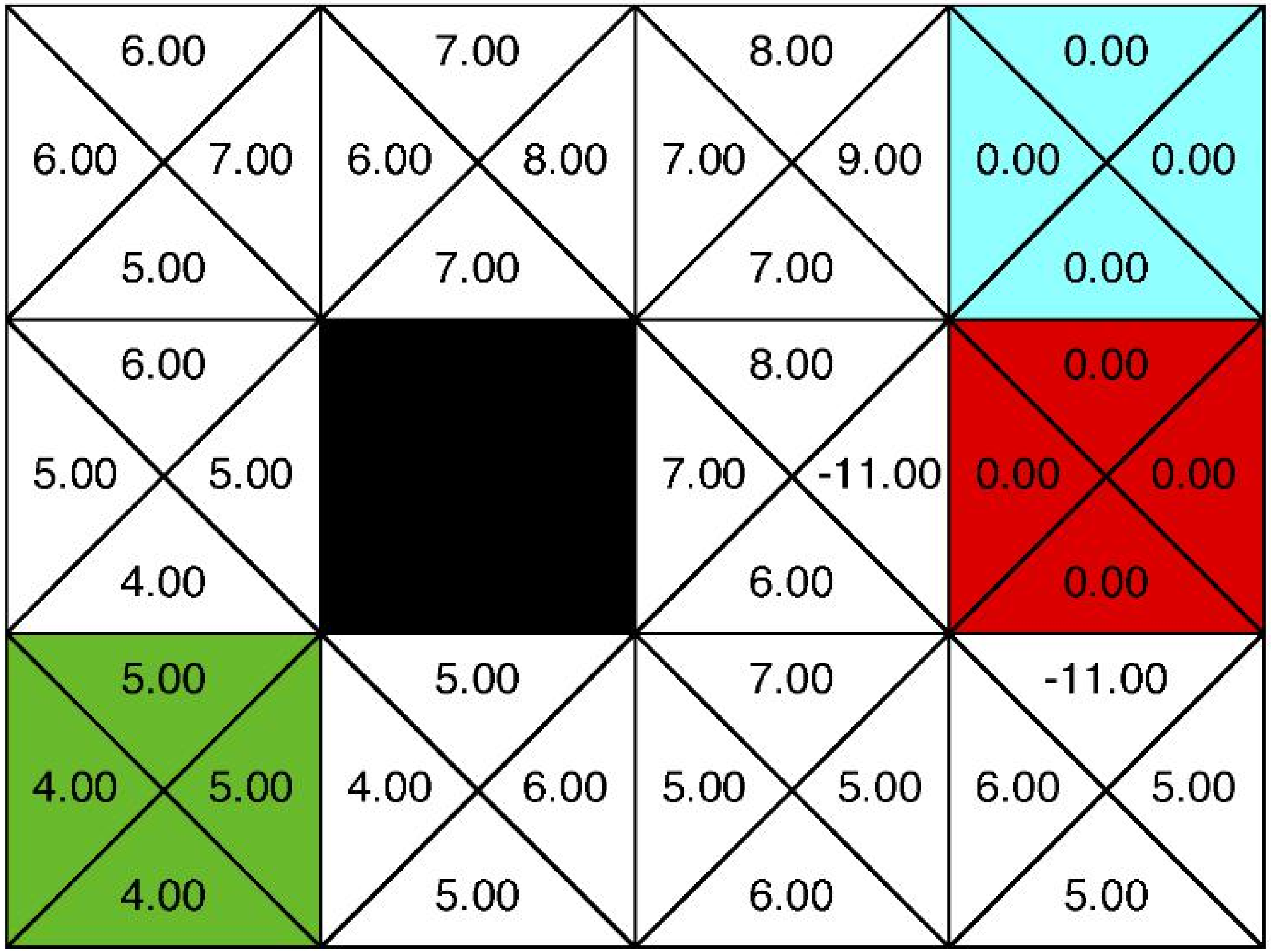


Image from [1]

Recap: $V-$ and $Q-$ values, converged ...

$\gamma = 1$, rewards $-1, +10, -10$, and no confusion - deterministic robot

7.00	8.00	9.00	10.00
6.00		8.00	-10.00
5.00	6.00	7.00	6.00



$$V(S_t) = R_{t+1} + V(S_{t+1})$$
$$Q(S_t, A_t) = R_{t+1} + \max_a Q(S_{t+1}, a)$$

Model-free TD learning, updating after each transition

- Observe, experience environment through learning episodes, collecting:

$$S_t, A_t, R_{t+1}, S_{t+1}, A_{t+1}, R_{t+2}, \ldots$$

- Update by mimicking Bellman updates after each transition $(S_t, A_t, R_{t+1}, S_{t+1})$

Recap: Bellman optimality equations for $v(s)$ and $q(s, a)$

$$
\begin{aligned}
v(s) &= \max_a \sum_{s'} p(s'|s,a)\left[r(s,a,s') + \gamma\, v(s')\right] \\
&= \max_a q(s,a)
\end{aligned}
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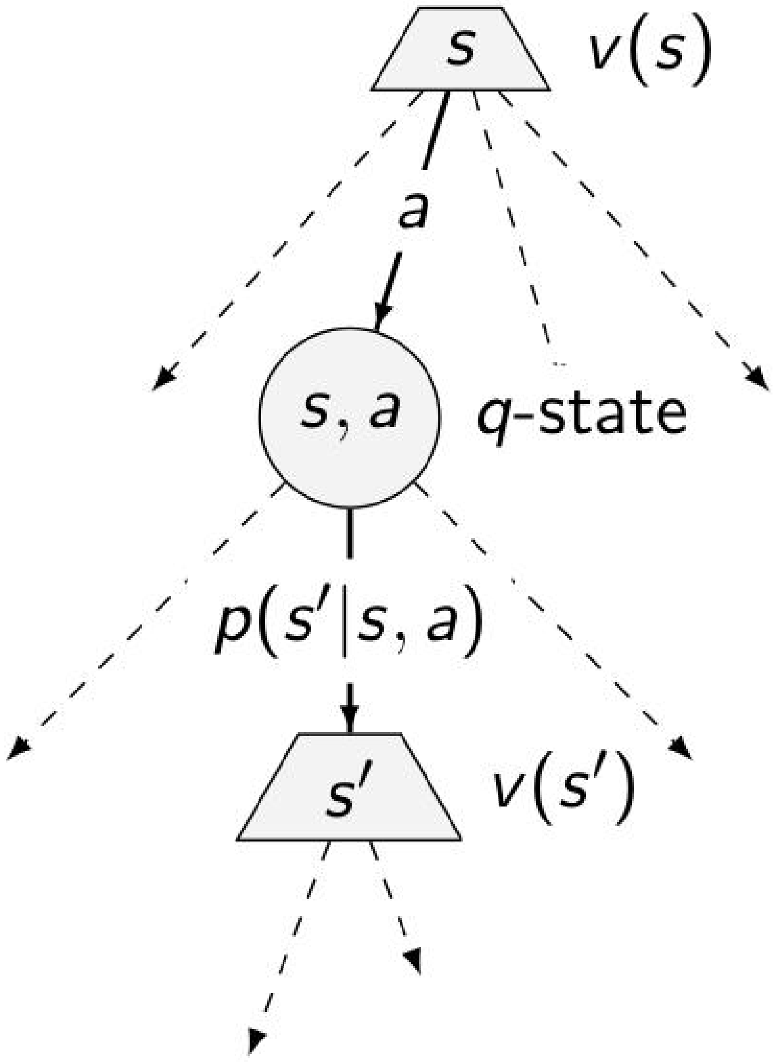

The value of a q-state (s, a):

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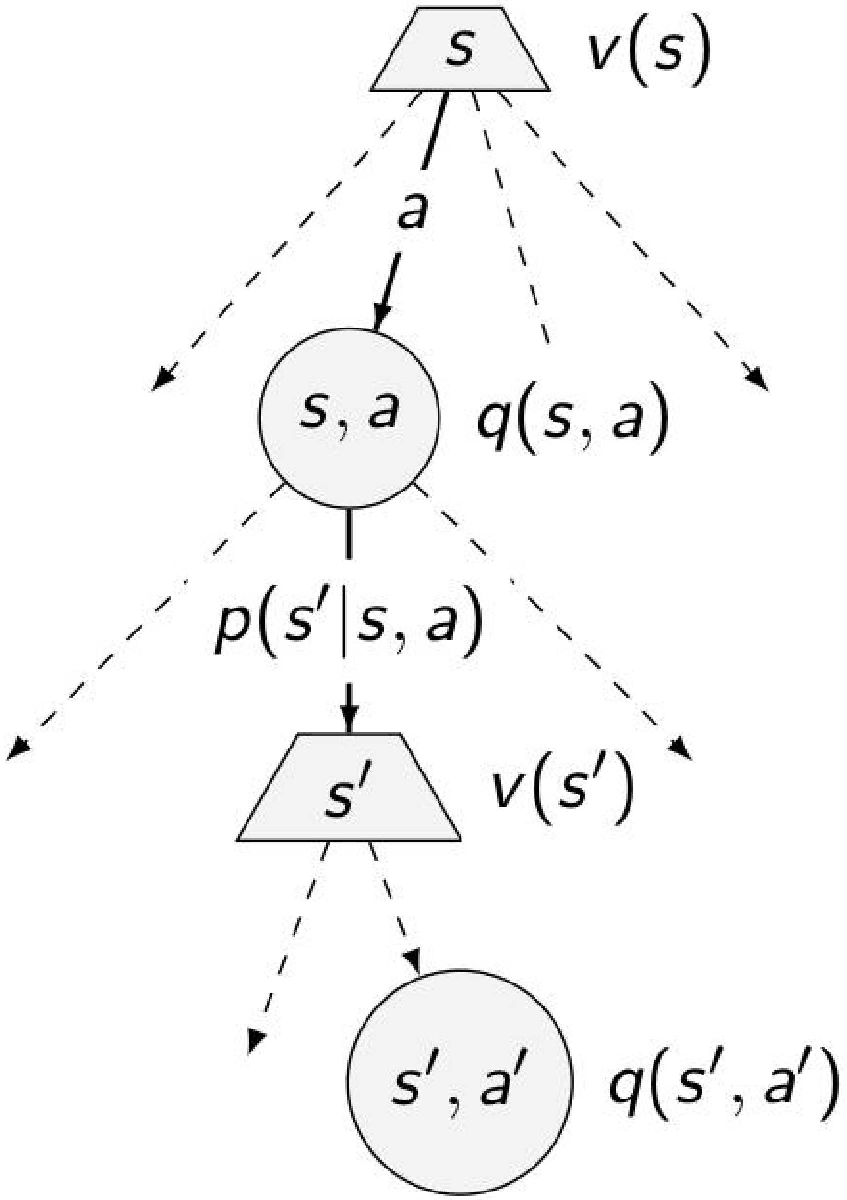

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Q-learning

Learn Q values as the robot/agent goes (temporal difference). If some Q quantity not known, initialize.

- time t, at S_t
- take $A_t \in \mathcal{A}(S_t)$, observe R_{t+1}, S_{t+1}
- compute trial/sample estimate at time t
  trial $= R_{t+1} + \gamma \max_a Q(S_{t+1}, a)$
- α temporal difference update
  $Q(S_t, A_t) \leftarrow Q(S_t, A_t) + \alpha(\text{trial} - Q(S_t, A_t))$
- $S_t \leftarrow S_{t+1}$ and repeat (unless S_t is terminal)

In each step Q approximates the optimal q^* function.

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Q-learning: algorithm

step size $0 < \alpha \leq 1$
initialize $Q(s, a)$ for all $s \in \mathcal{S}, a \in \mathcal{S}(s)$
repeat episodes:
    initialize S
    for for each step of episode: **do**
        choose A from S
        take action A, observe R, S'
        $Q(S, A) \leftarrow Q(S, A) + \alpha\big[R + \gamma \max_a Q(S', a) - Q(S, A)\big]$
        $S \leftarrow S'$
    end for until S is terminal
until Time is up, ...

How to select A_t in S_t?

- time t, at S_t
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How to select A_t in S_t?

- time t, at S_t
- take A_t derived from Q , observe R_{t+1}, S_{t+1}
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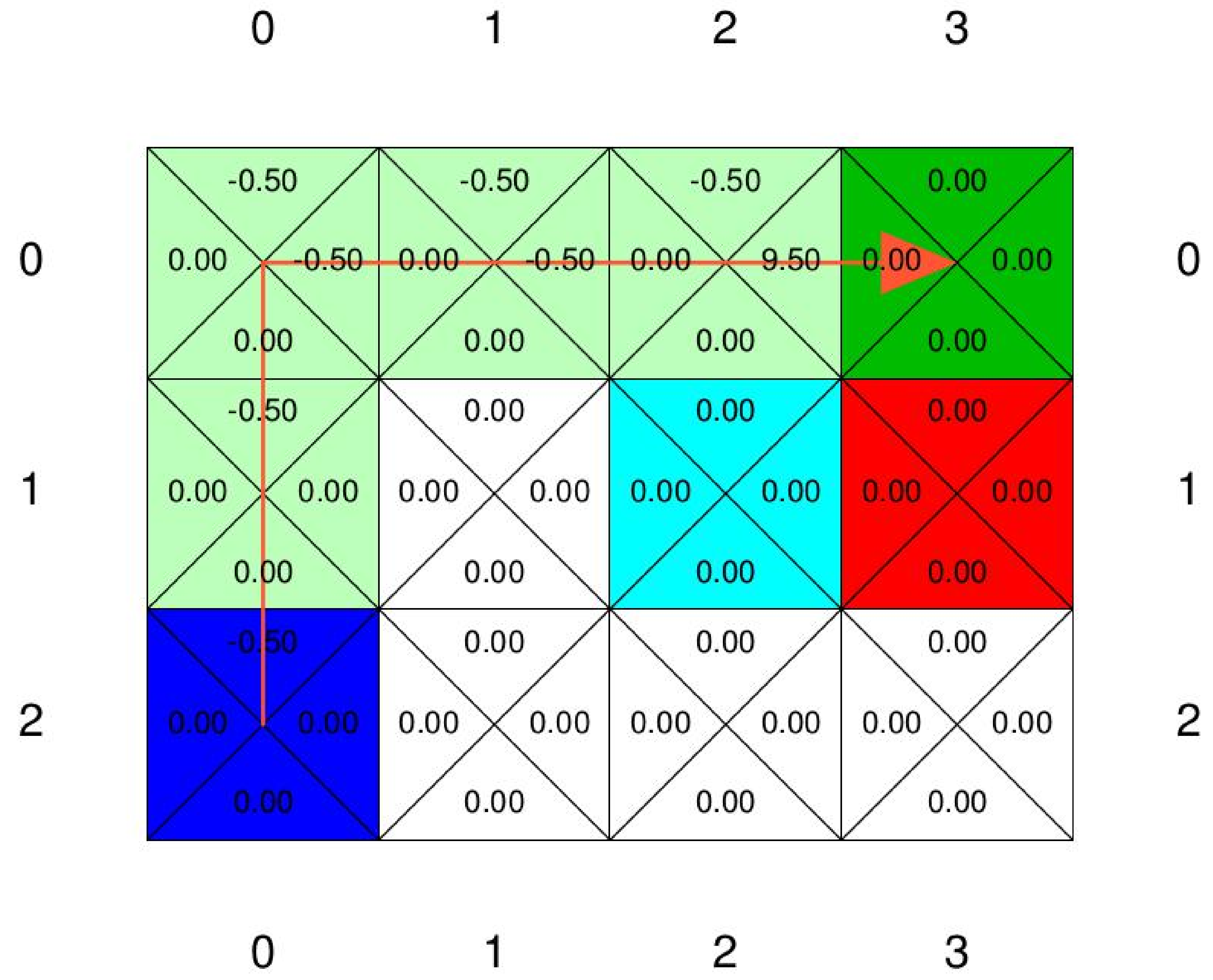
$\ldots A_t$ derived from Q

What about keeping optimality, taking max?

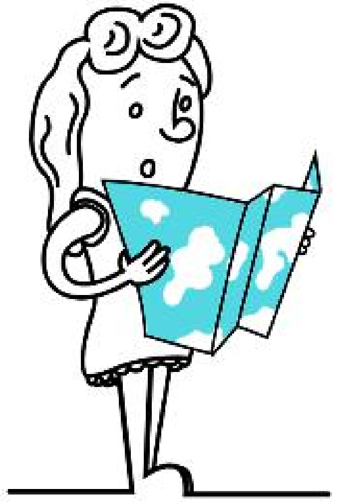
$$A_t = \arg\max_a Q(S_t, a)$$

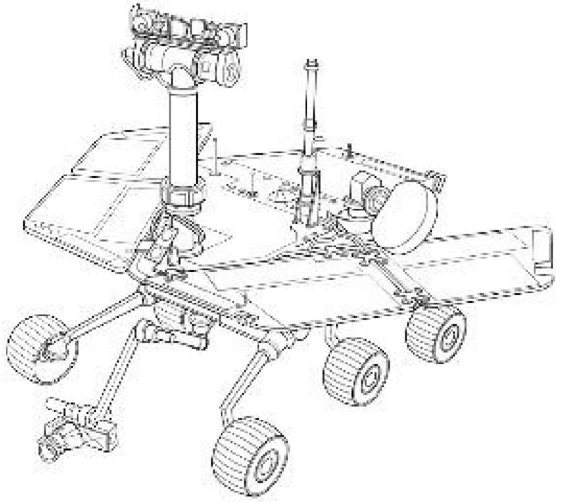
see the demo run of `rl_agents.py`.

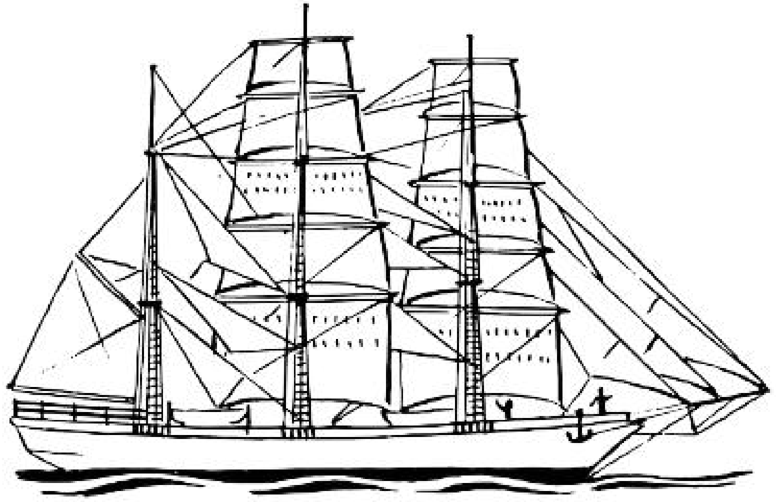
Two good goal states



Exploration vs Exploitation







- Drive the known road or try a new one?
- Go to the university menza or try a nearby restaurant?
- Use the SW (operating system) I know or try new one?
- Go to bussiness or study a demanding program?
- ...

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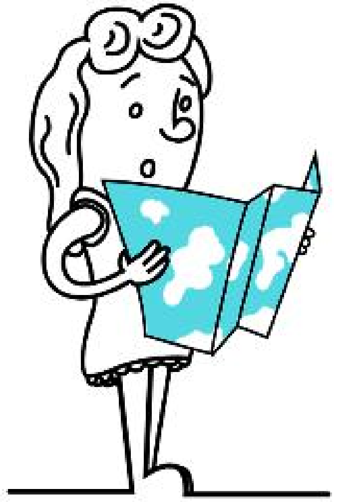

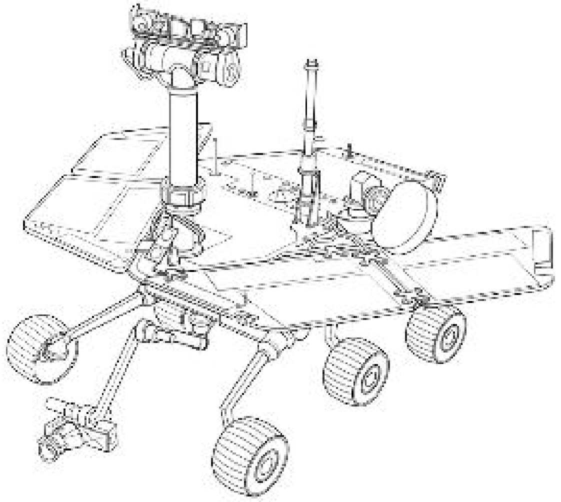

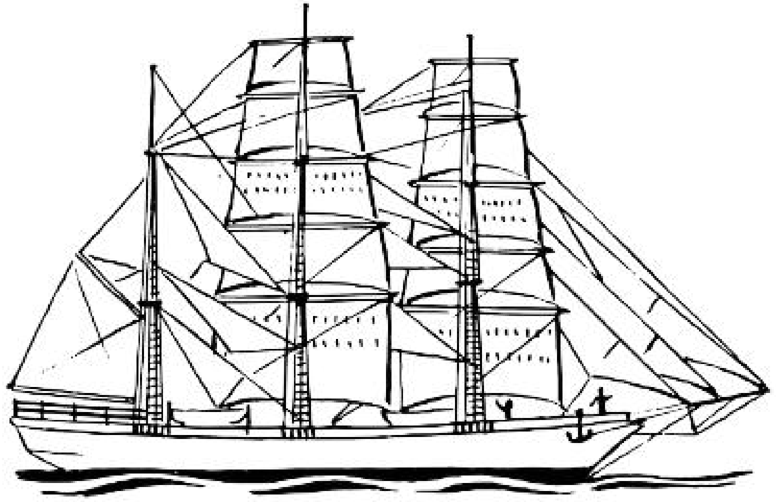

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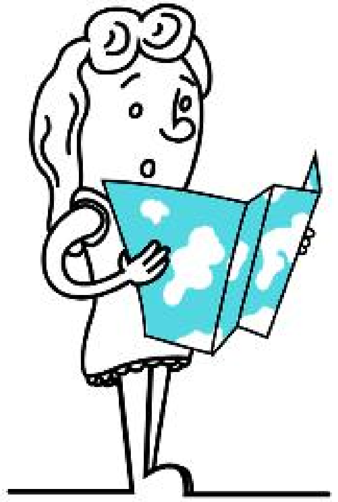
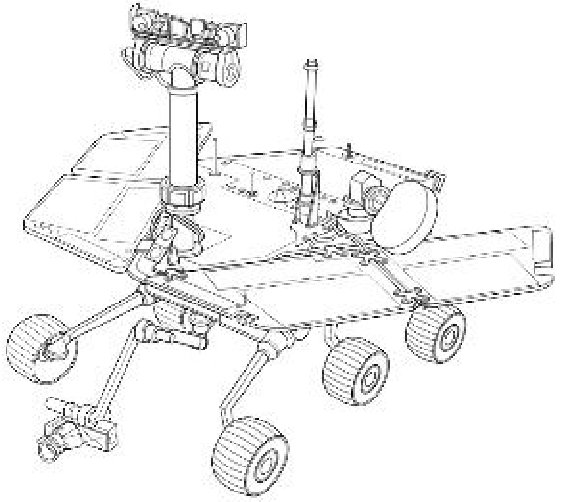
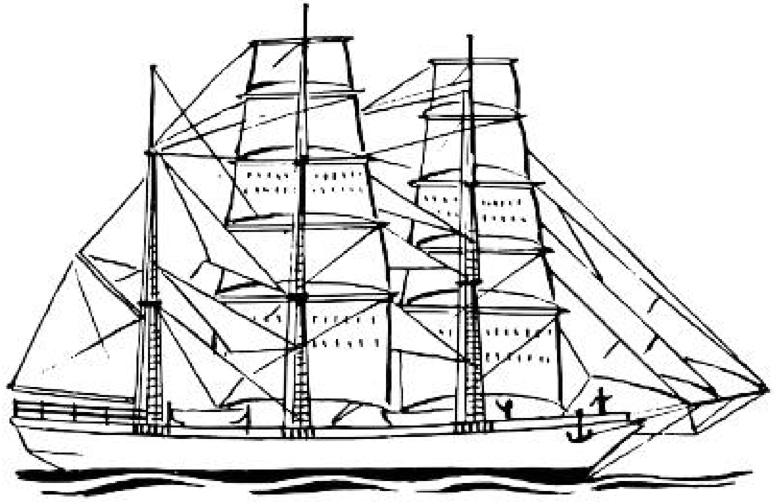

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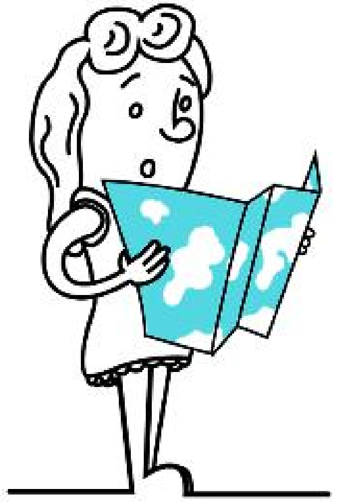
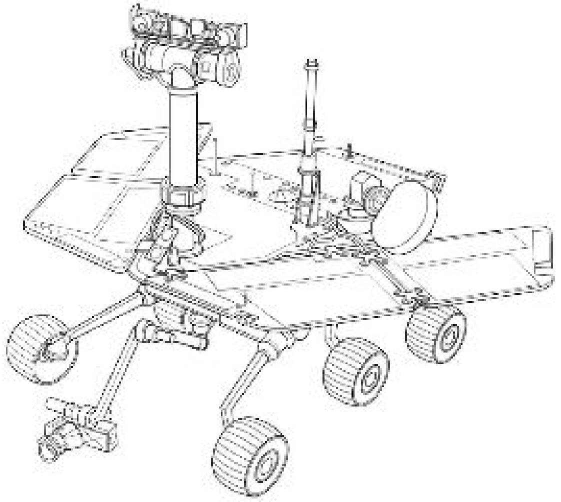
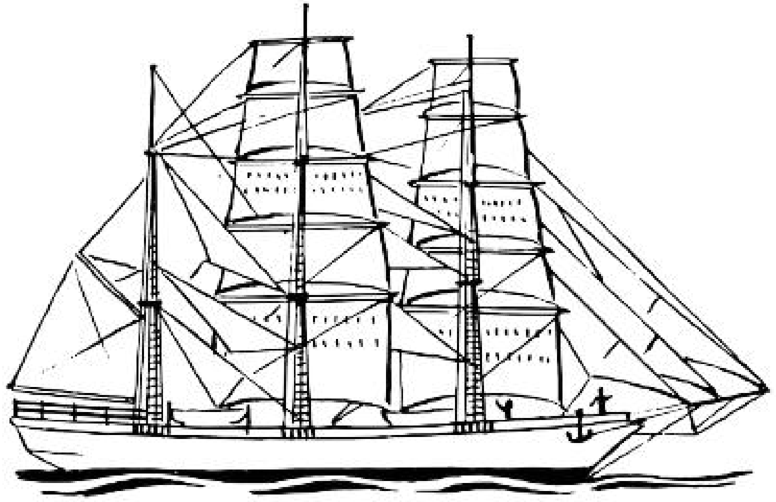

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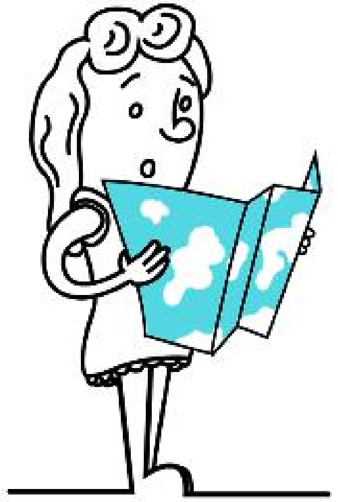

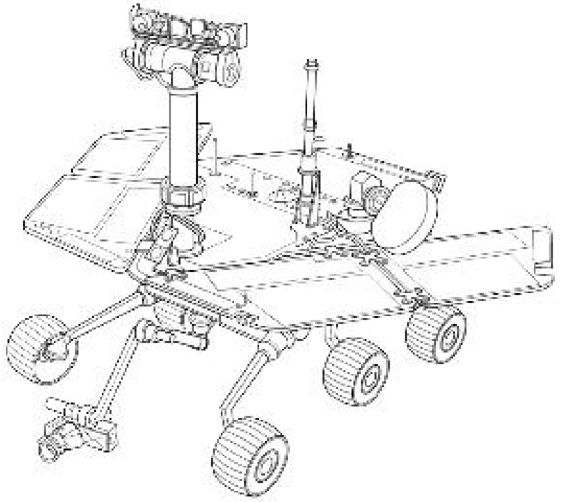

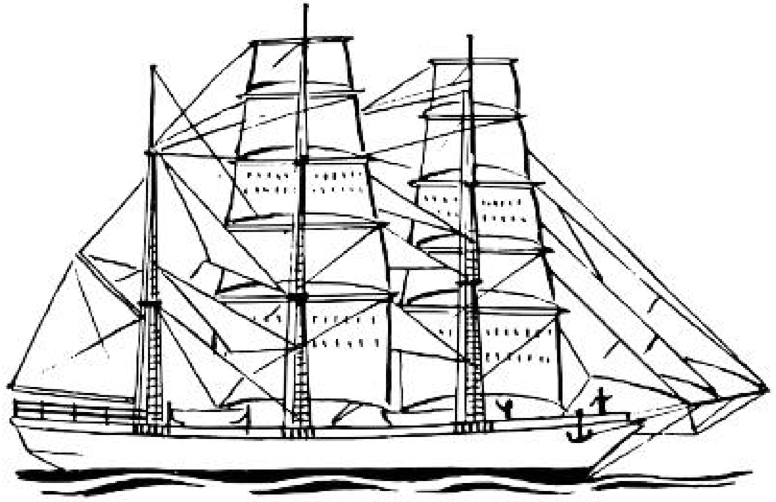

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How to explore?

Random (ϵ-greedy):

- Flip a coin every step.
- With probability ϵ, act randomly.
- With probability $1 - \epsilon$, use the policy.

Problems with randomness?

- Keeps exploring forever.
- Should we keep ϵ fixed (over learning)?
- ϵ same everywhere?

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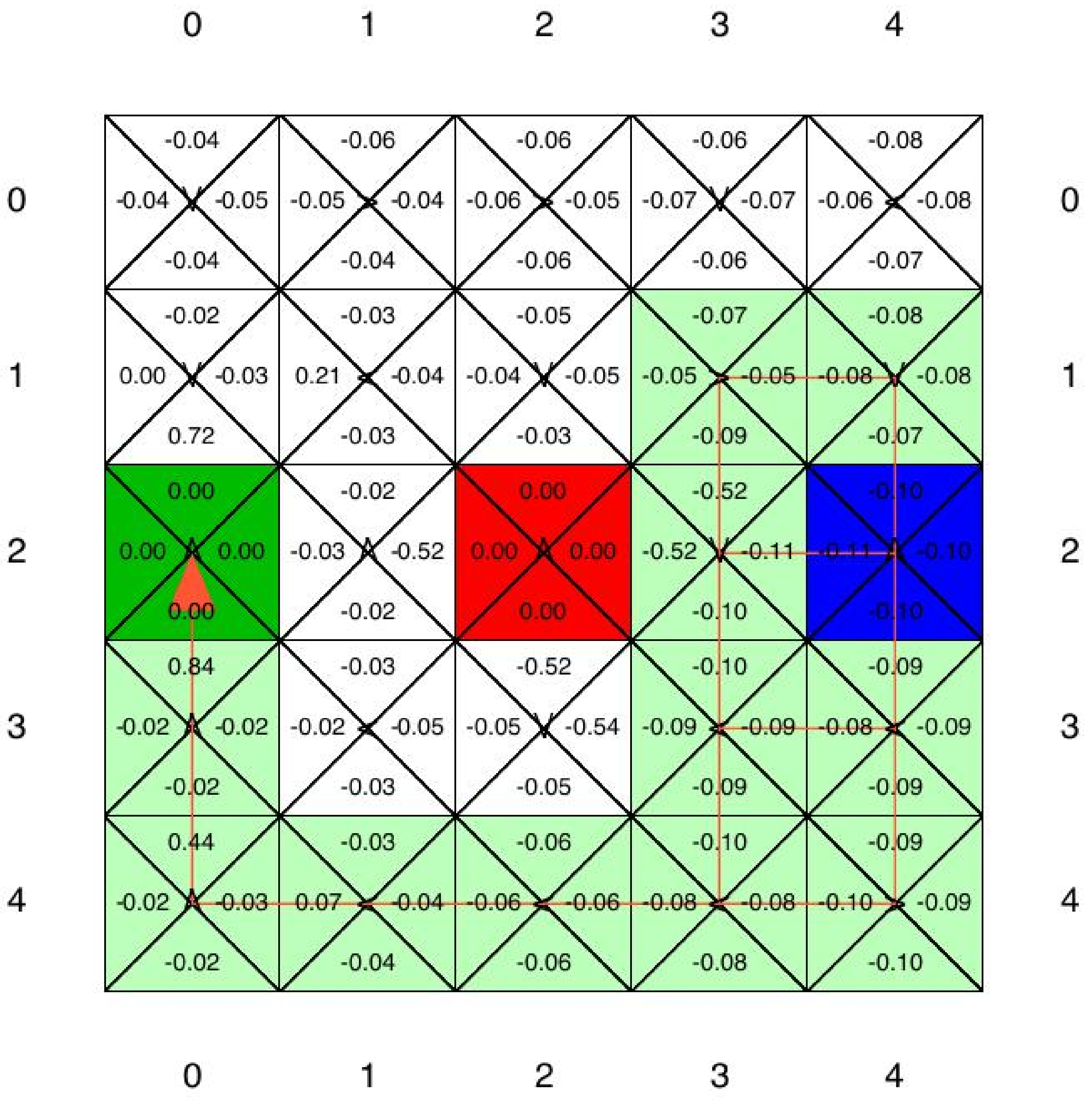

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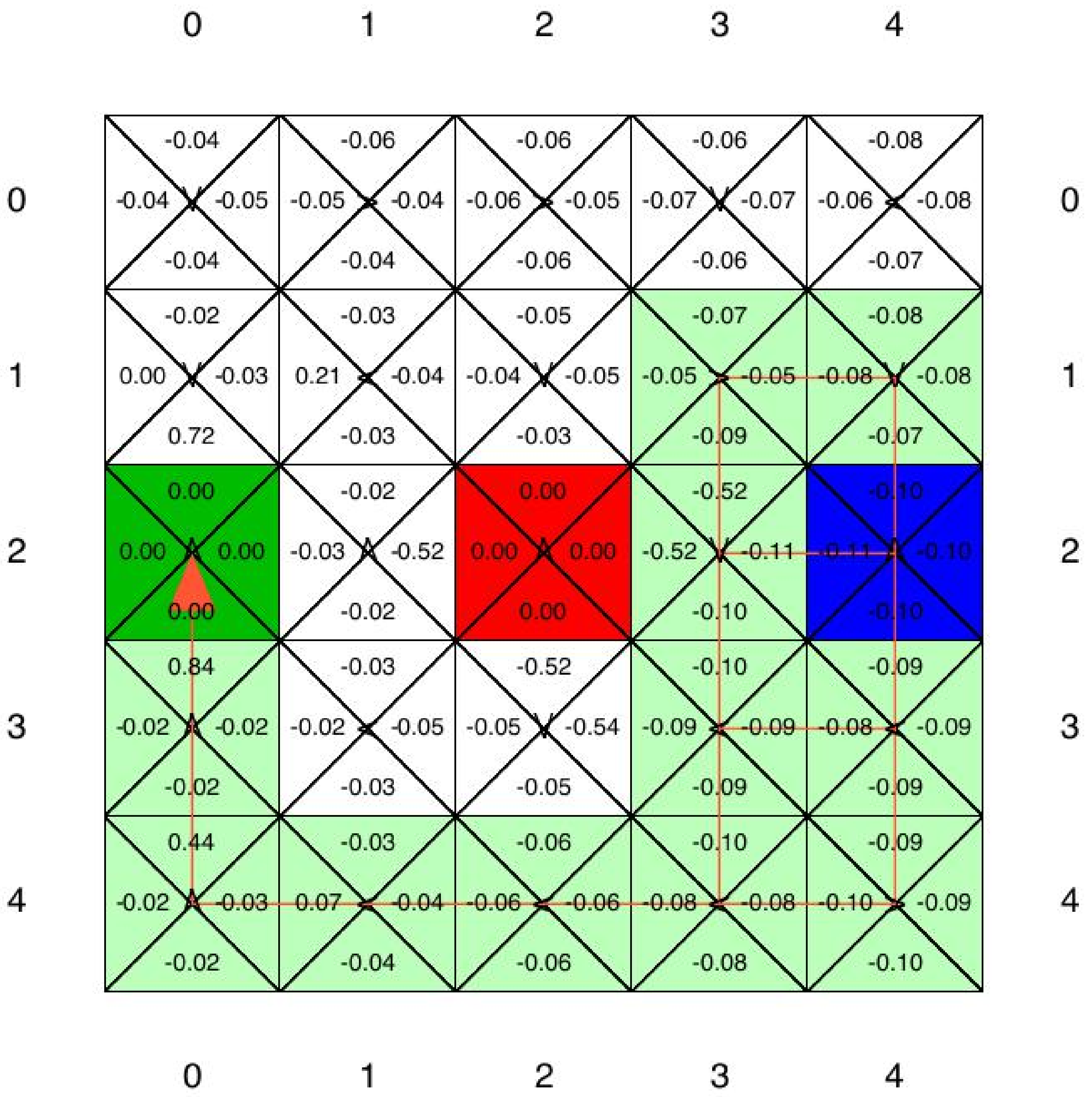

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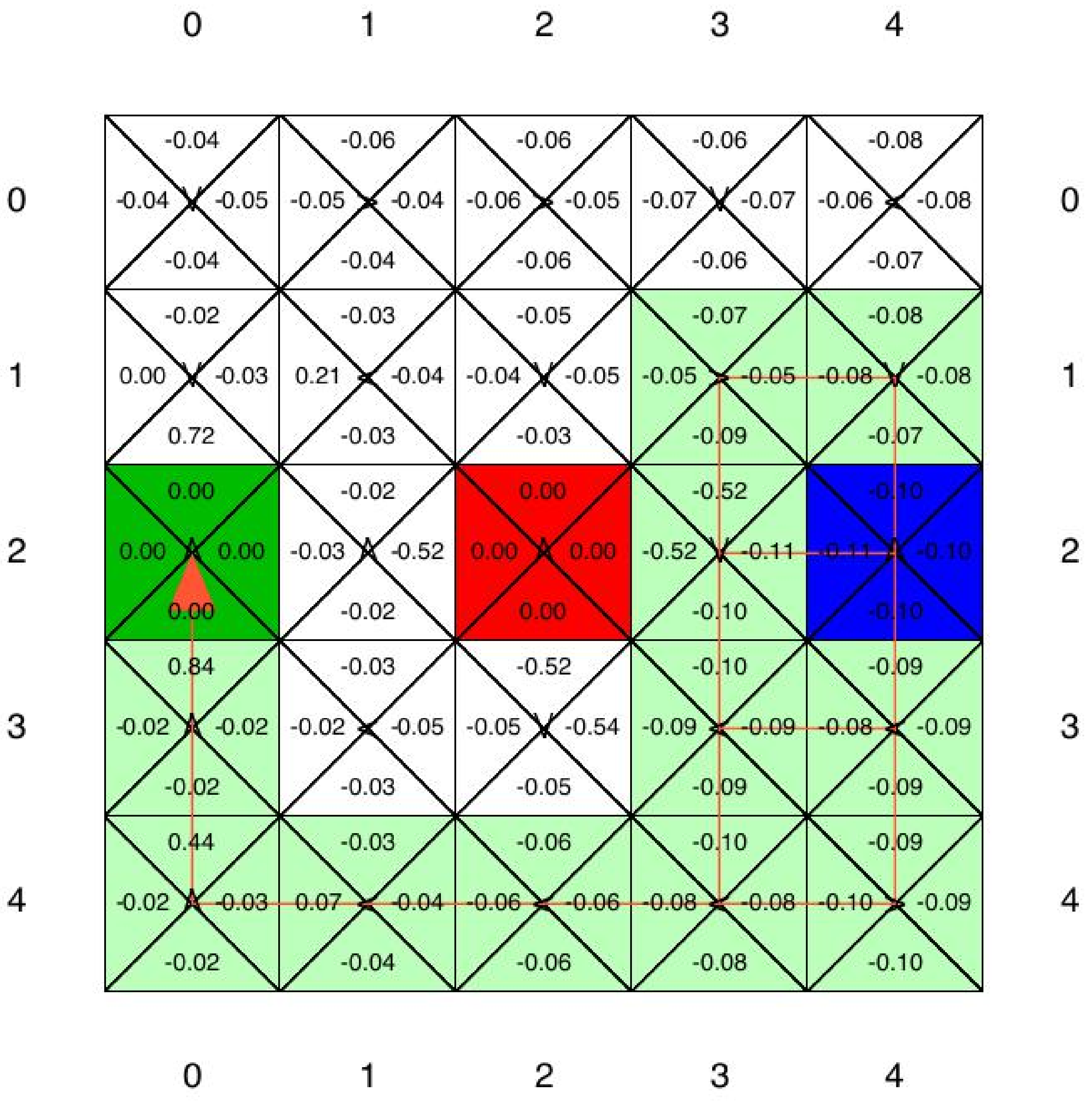

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Exploration function $f(u, n)$

- Regular trial/sample estimate: trial $= R_{t+1} + \gamma \max_a Q(S_{t+1}, a)$
- If (S_t, a) not yet tried, then perhaps too pesimistic.
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where $f(u, n)$

$$
\begin{aligned}
f(u, n) &= R^{+} \text{ if } n < N_e \\
&= u \text{ otherwise}
\end{aligned}
$$

where

- R^{+} is an optimistic estimate of the best possible reward obtainable in any state
- N_e fixed parameter
- The function $f(u, n)$ should be increasing in u and decreasing in n.

Exploration function $f(u, n)$

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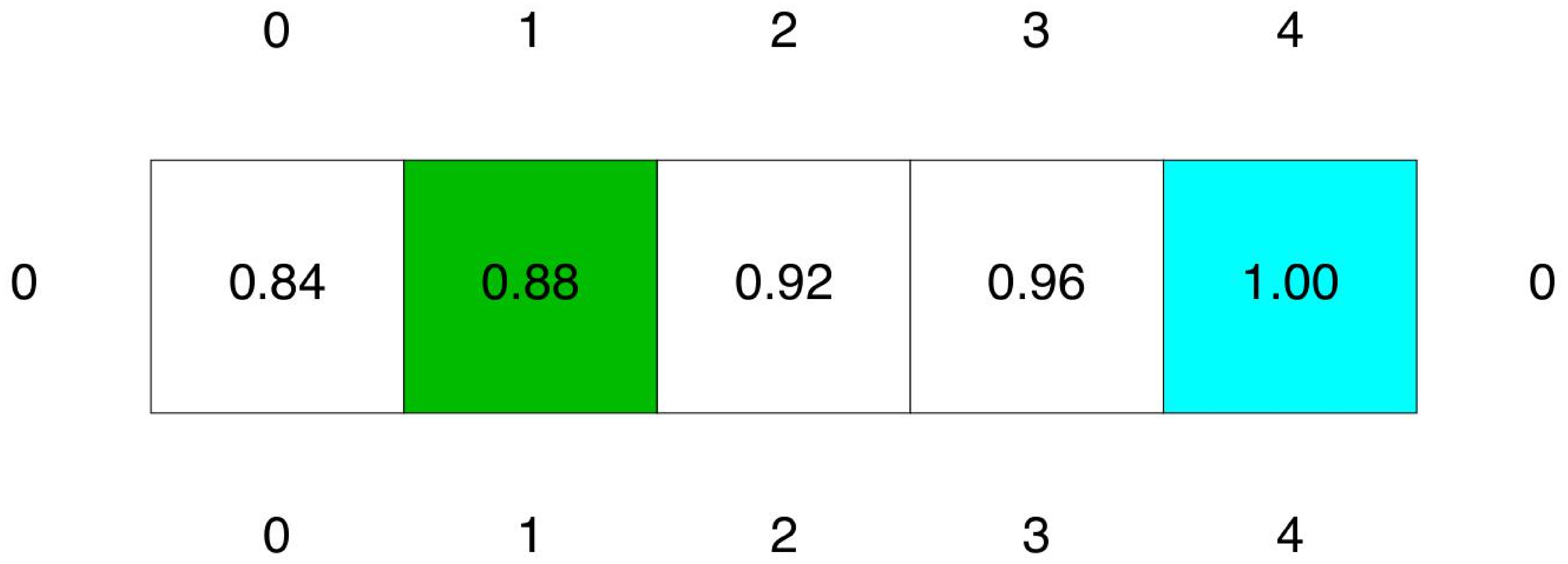
Going beyond tables - generalizing across states



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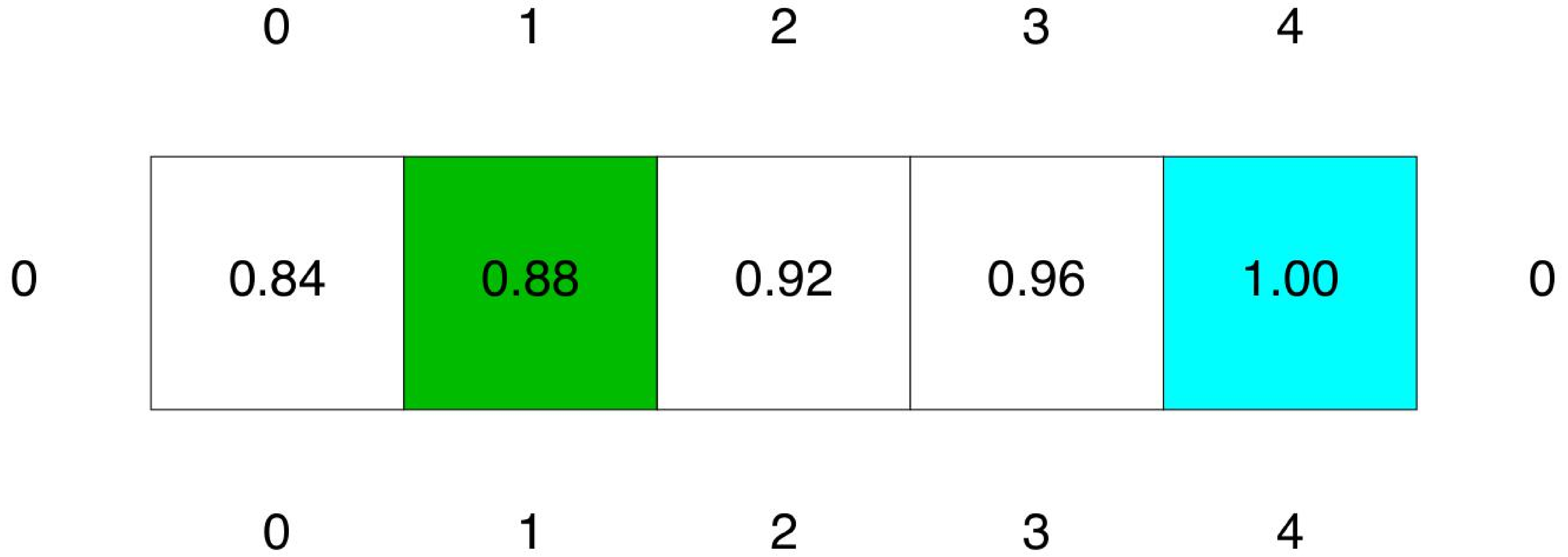
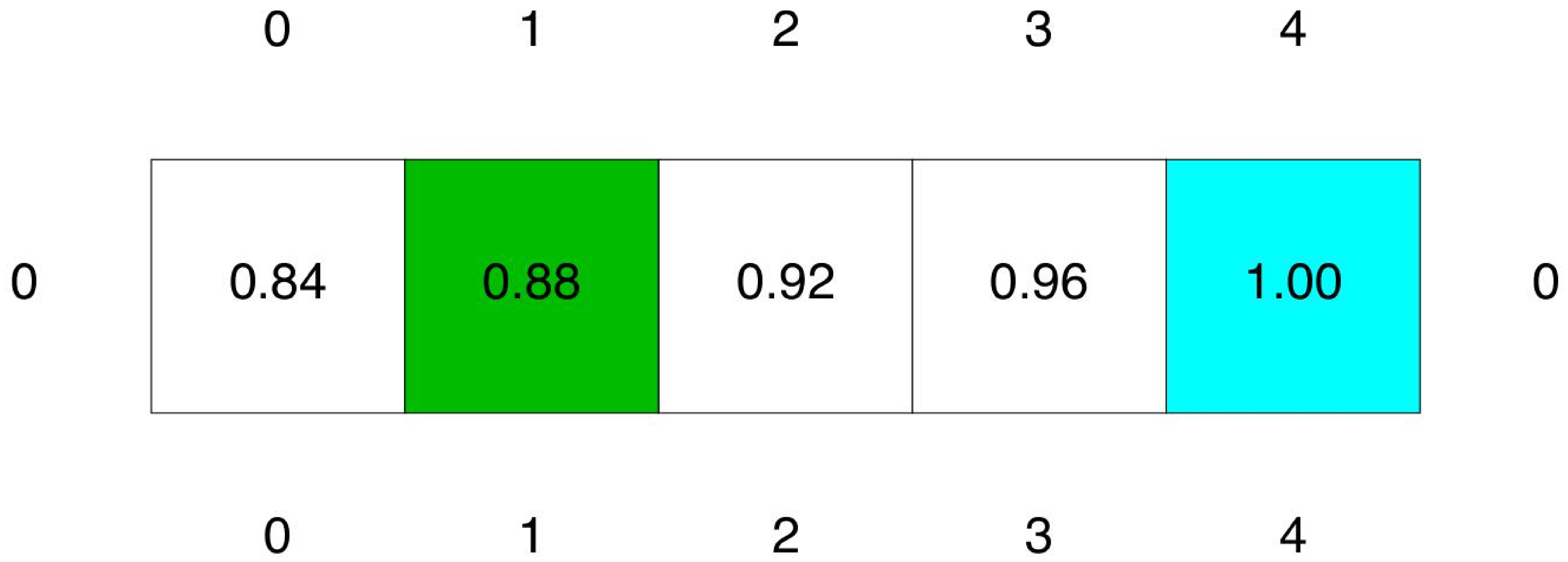


Going beyond tables - generalizing across states

	0	1	2	3	4	
0	0.84	0.80	0.76	0.72	0.68	0
1	0.88	0.84	0.80	0.76	0.72	1
2	0.92	0.88	0.84	0.80	0.76	2
3	0.96	0.92	0.88	0.84	0.80	3
4	1.00	0.96	0.92	0.88	0.84	4
	0	1	2	3	4	

v(s) not as table but as an approximation function



$$\hat{v}(s, \mathbf{w}) = w_0 + w_1 s$$

What are w_0, w_1 equal to?

Instead of the complete table, only 2 parameters to learn $\mathbf{w} = [w_0, w_1]^\top$

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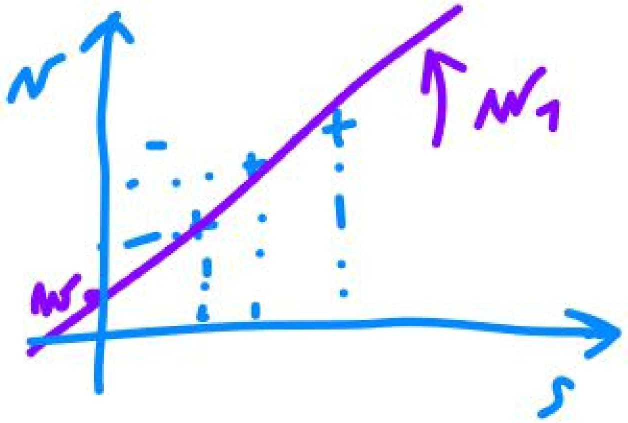

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Linear value functions

7.00	8.00	9.00	10.00
6.00		8.00	-10.00
5.00	6.00	7.00	6.00

$$
\begin{aligned}
\hat{v}(s, \mathbf{w}) &= w_1 f_1(s) + w_2 f_2(s) + w_3 f_3(s) + \cdots + w_n f_n(s) \\
\hat{q}(s, a, \mathbf{w}) &= w_1 f_1(s, a) + w_2 f_2(s, a) + w_3 f_3(s, a) + \cdots + w_n f_n(s, a)
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Learning **w** by Stochastic Gradient Descent (SGD)

- assume $\hat{v}(s, \mathbf{w})$ differentiable in all states
- we update $\mathbf{w}$ in discrete time steps t
- in each step S_t we observe a new example of (true) $v^\pi(S_t)$
- $\hat{v}(S_t, \mathbf{w})$ is an approximator $\rightarrow$ error $= v^\pi(S_t) - \hat{v}(S_t, \mathbf{w}_t)$

$$\begin{aligned}
\mathbf{w}_{t+1} &\doteq \mathbf{w}_t - \frac{1}{2}\alpha \nabla \Big[v^\pi(S_t) - \hat{v}(S_t, \mathbf{w}_t) \Big]^2 \\
&= \mathbf{w}_t + \alpha \Big[v^\pi(S_t) - \hat{v}(S_t, \mathbf{w}_t) \Big] \nabla \hat{v}(S_t, \mathbf{w}_t)
\end{aligned}$$

$$\nabla f(\mathbf{w}) \doteq \left[\frac{\partial f(\mathbf{w})}{\partial w_1}, \frac{\partial f(\mathbf{w})}{\partial w_2}, \ldots, \frac{\partial f(\mathbf{w})}{\partial w_d} \right]^\top$$

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Approximate Q-learning (of a linear combination)

$$\hat{q}(s, a, \mathbf{w}) = w_1 f_1(s, a) + w_2 f_2(s, a) + w_3 f_3(s, a) + \cdots + w_n f_n(s, a)$$

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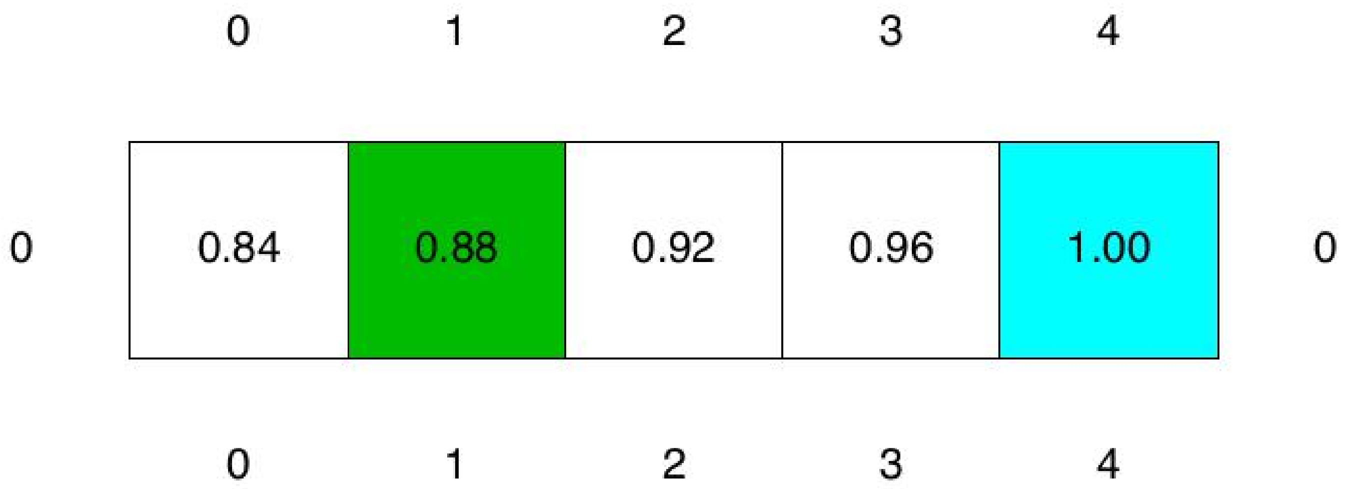
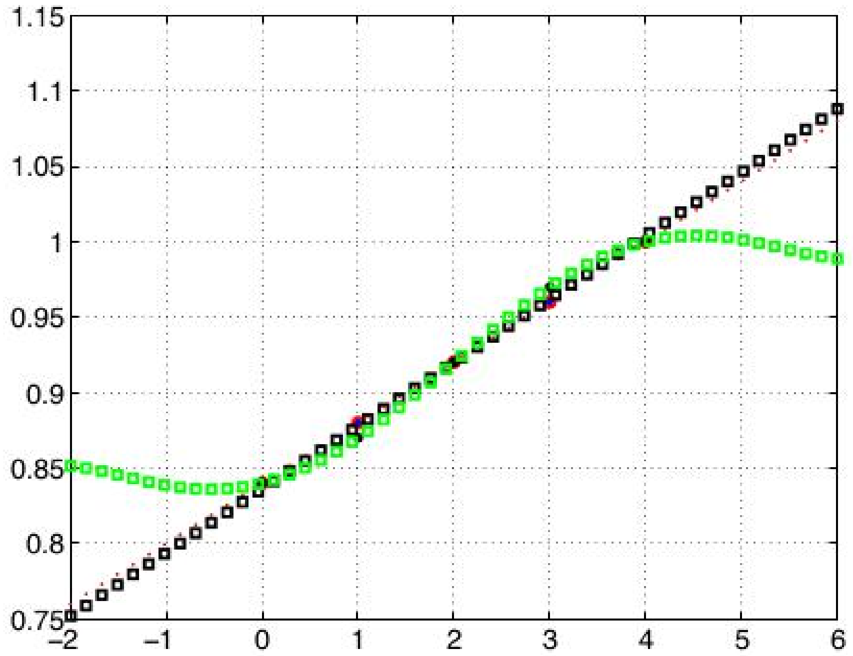
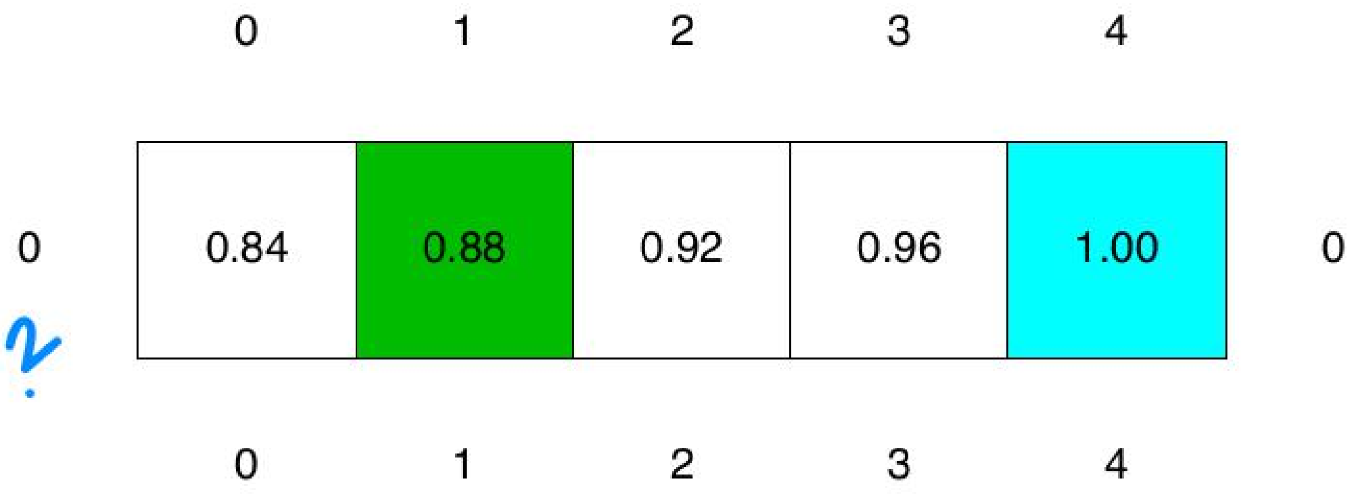
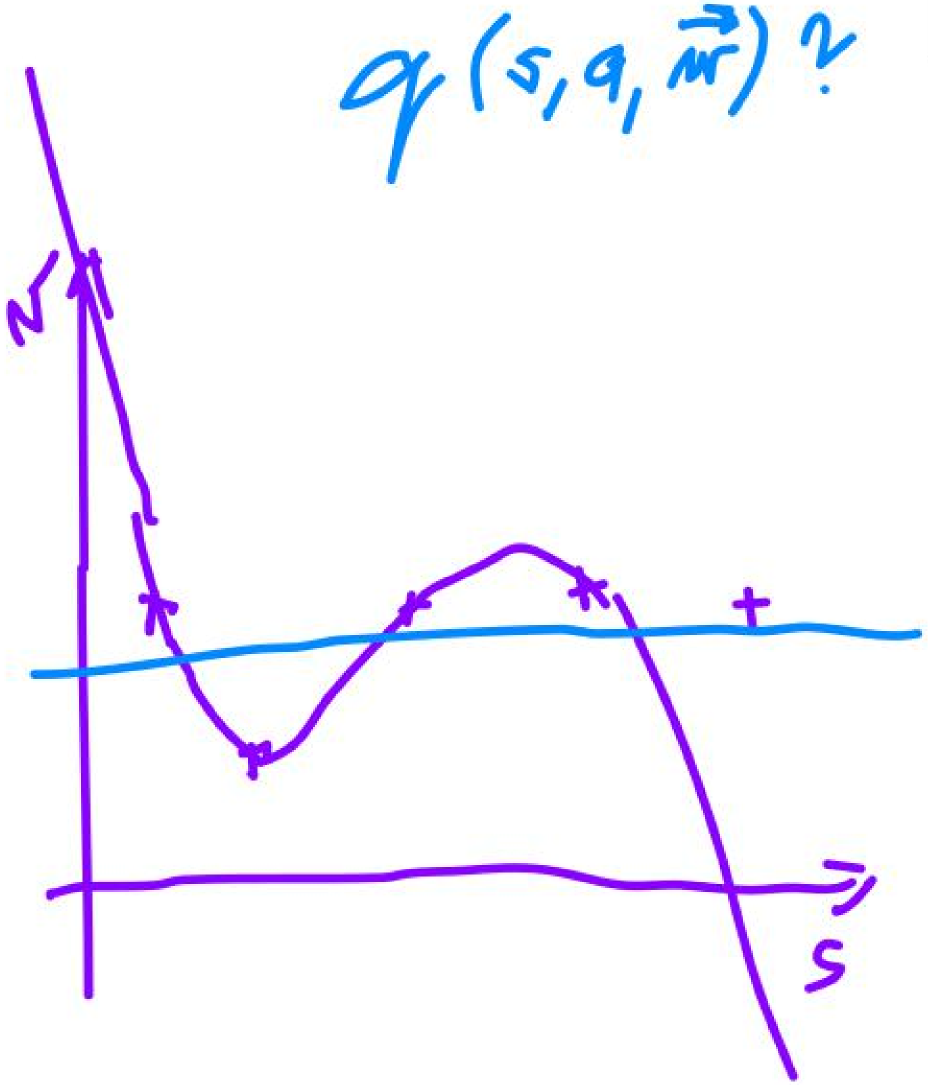
Overfitting

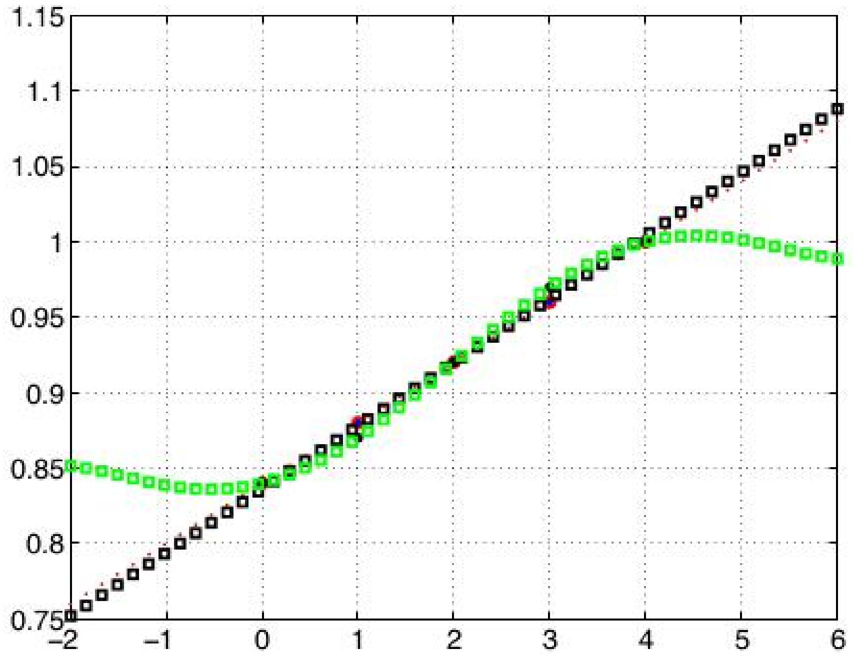




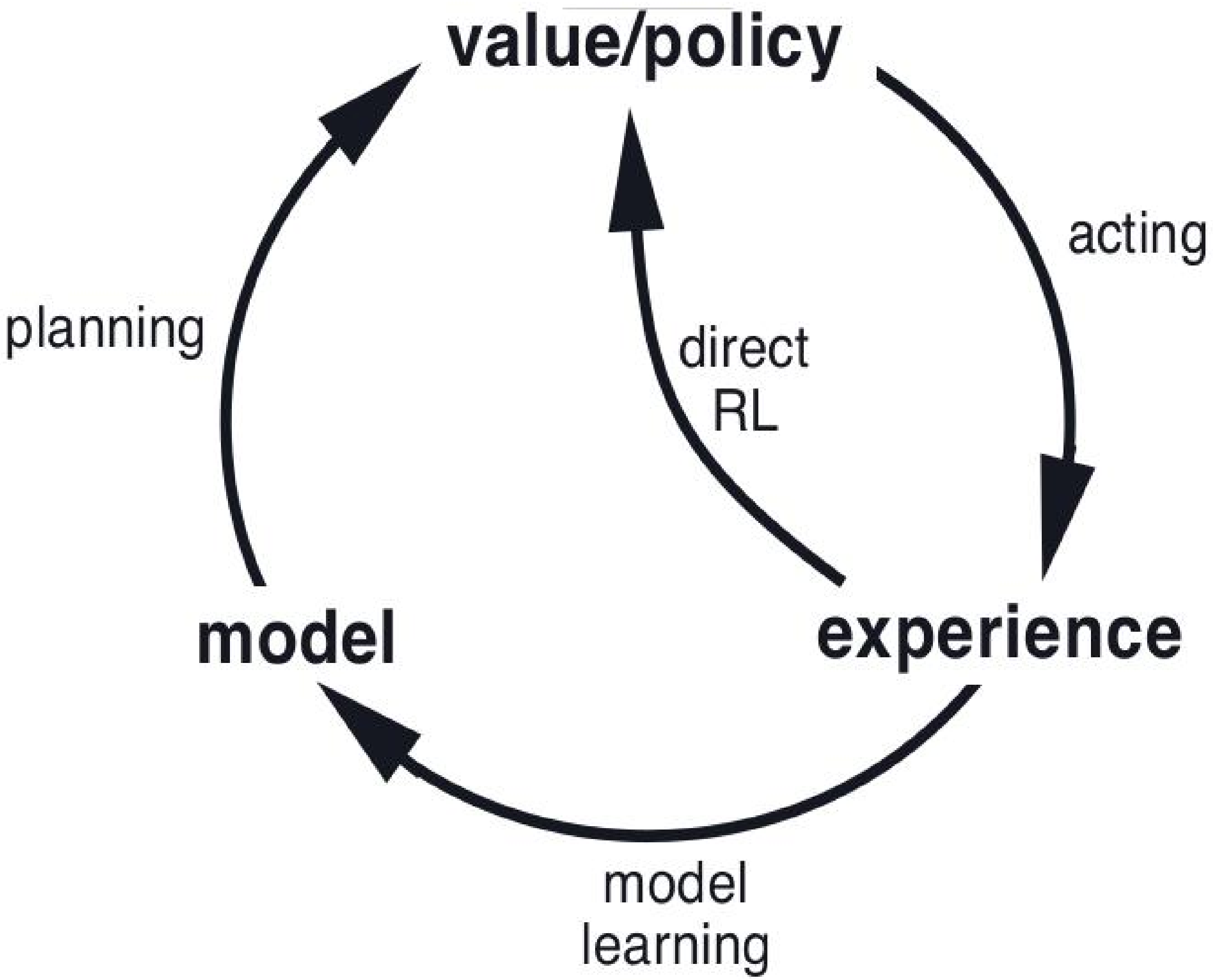
Overfitting

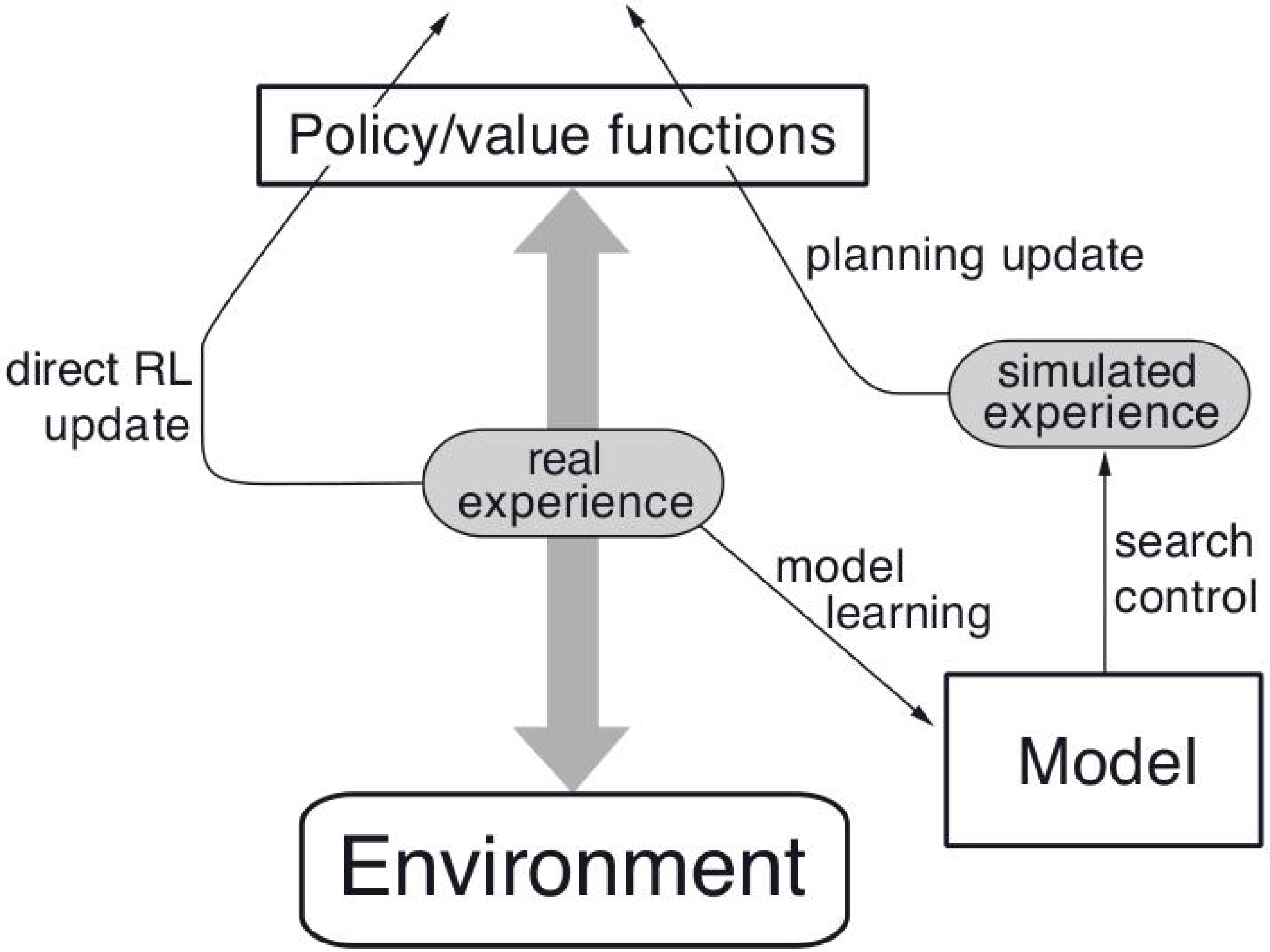






Going beyond - Dyna-Q integration planning, acting, learning





2

[2]Schemes from [2]

Overfitting

	0	1	2	3	4	
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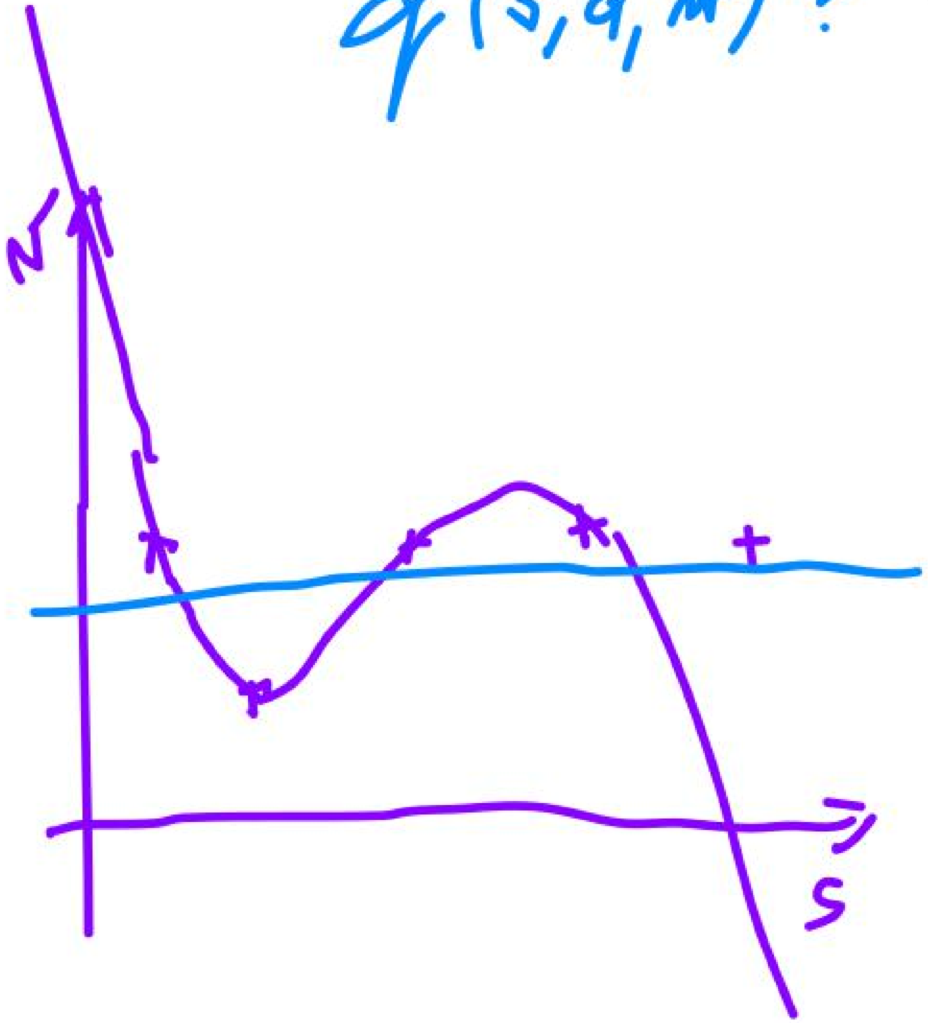
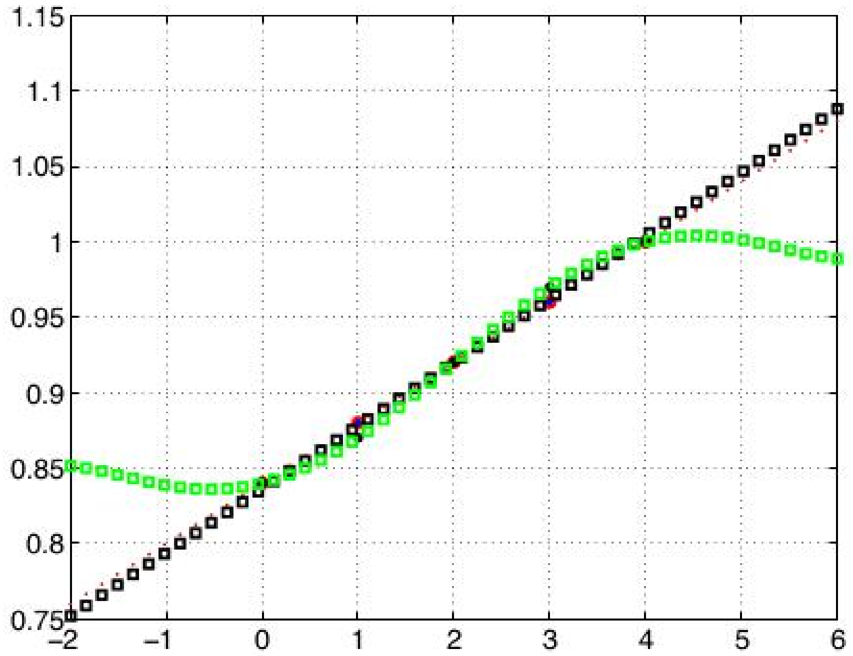
$q(s, a, \vec{w})$?





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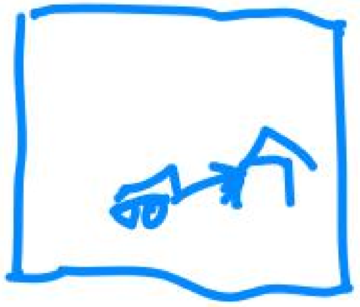

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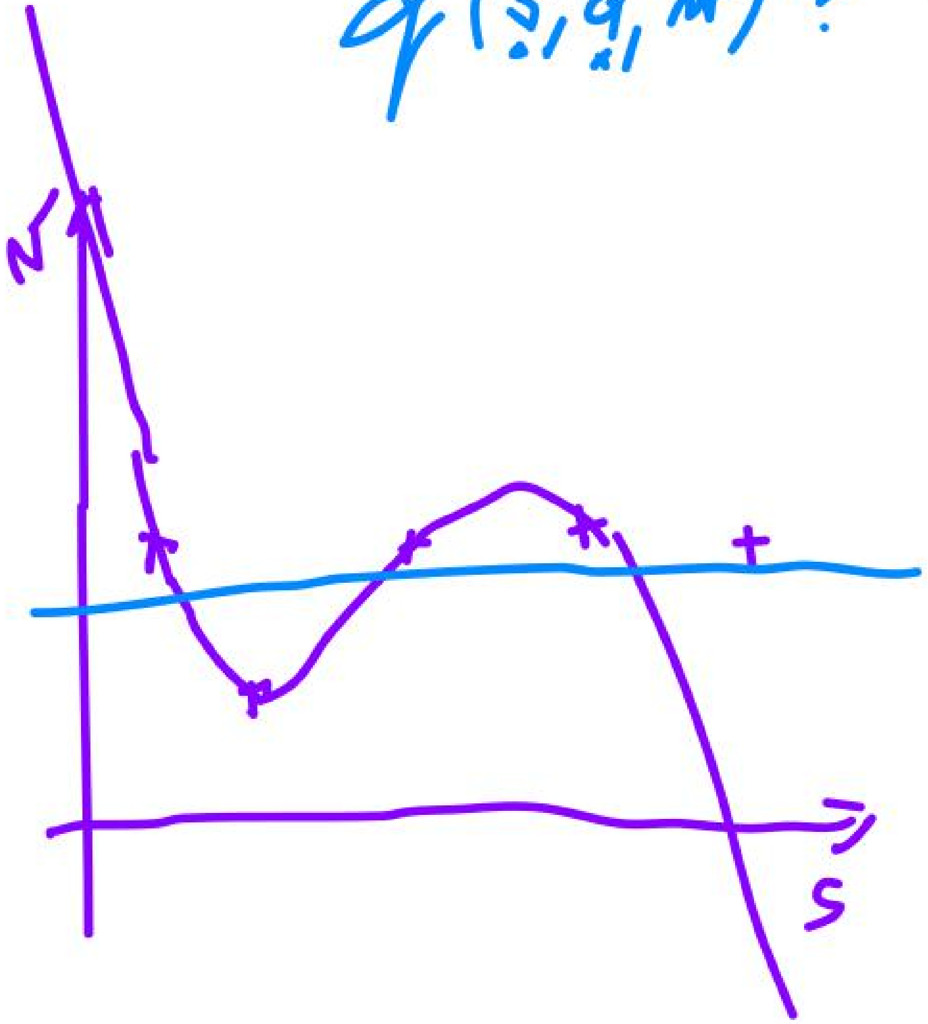

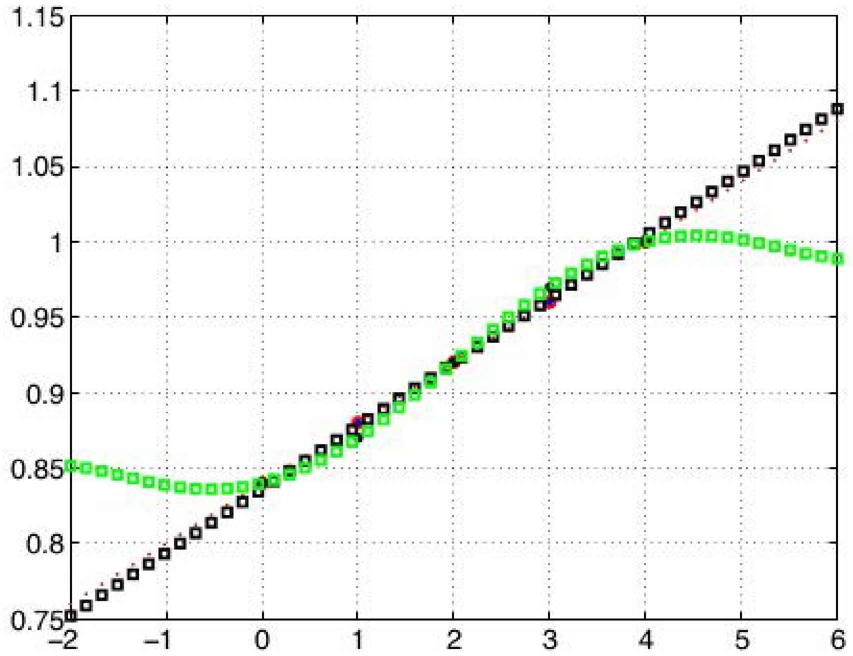

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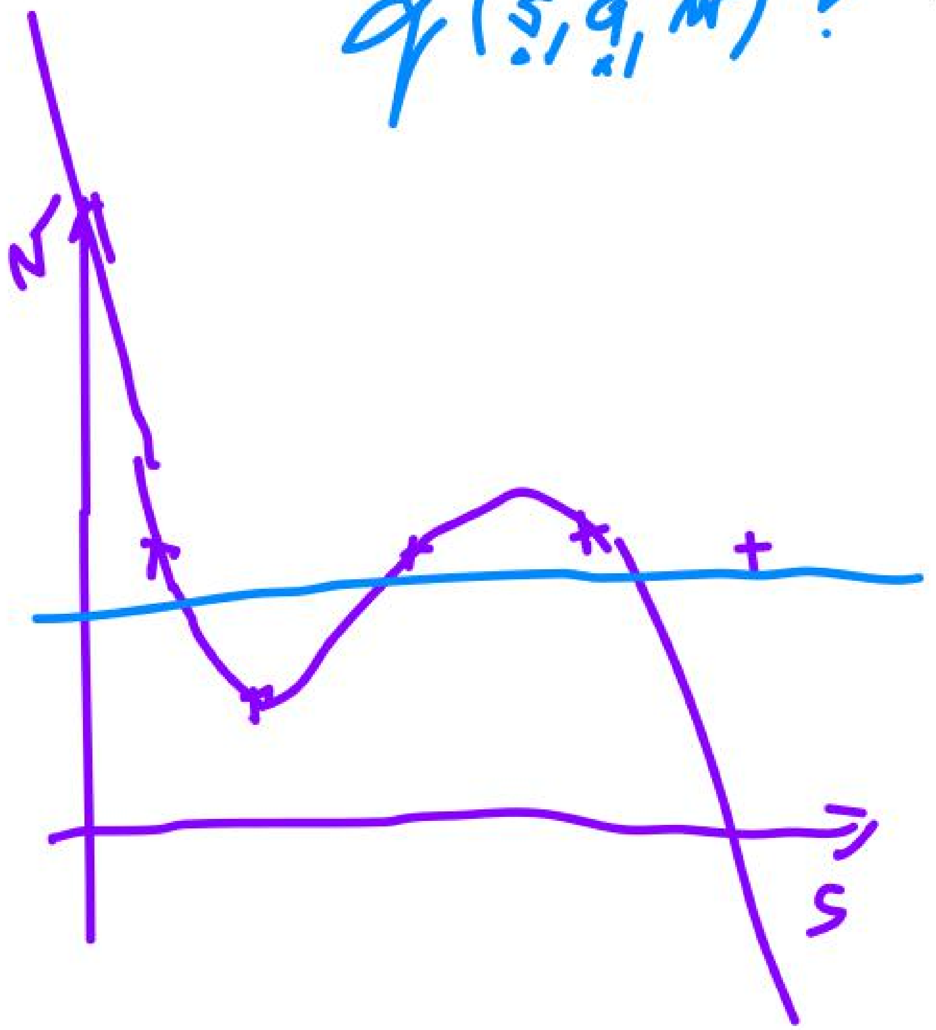
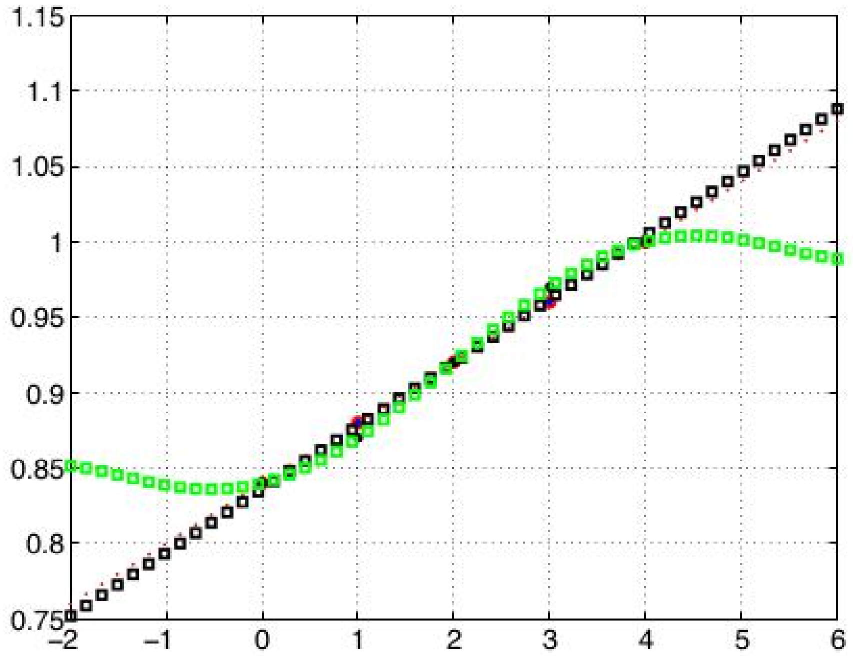
Overfitting

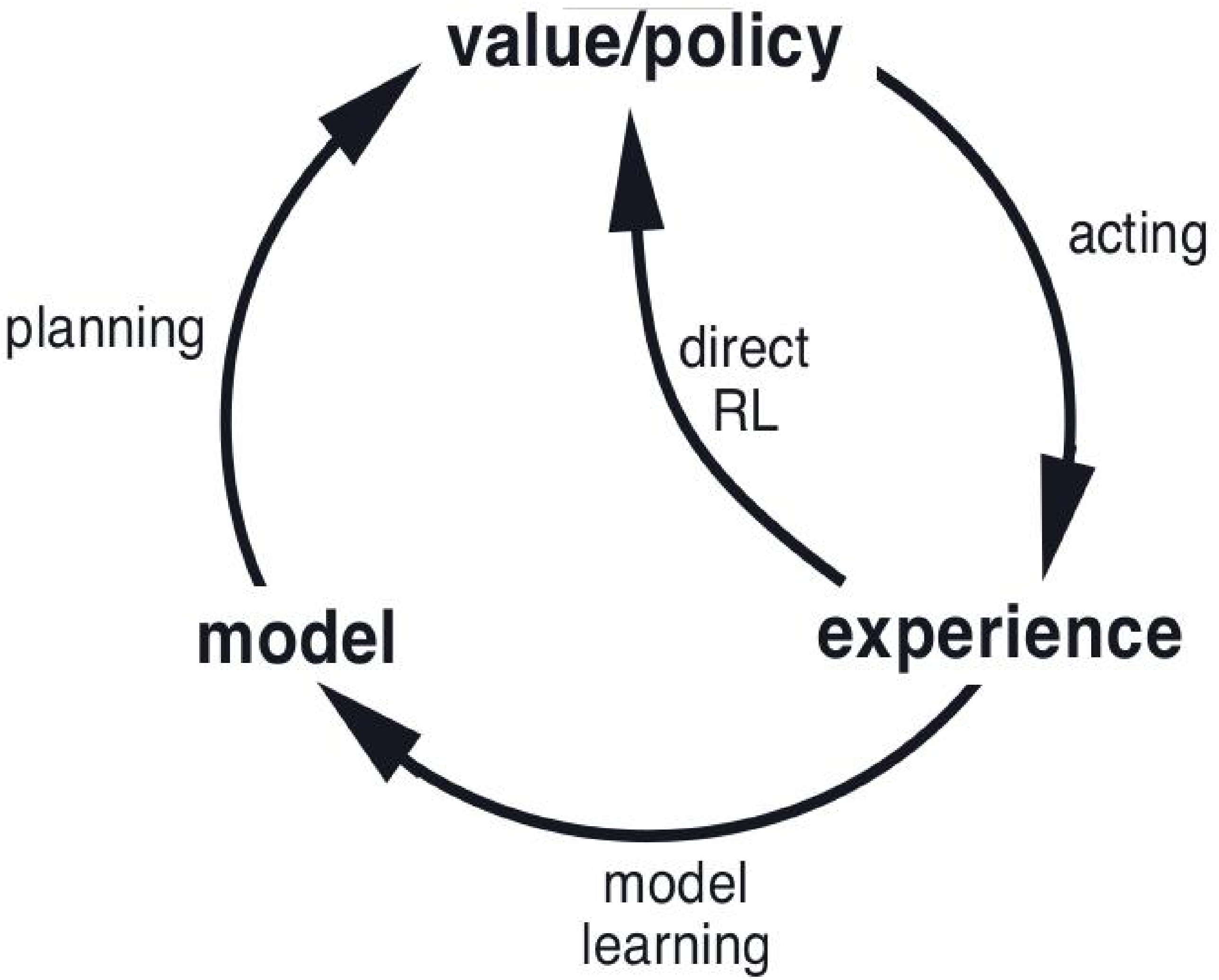
	0	1	2	3	4	
0	0.84	0.88	0.92	0.96	1.00	0

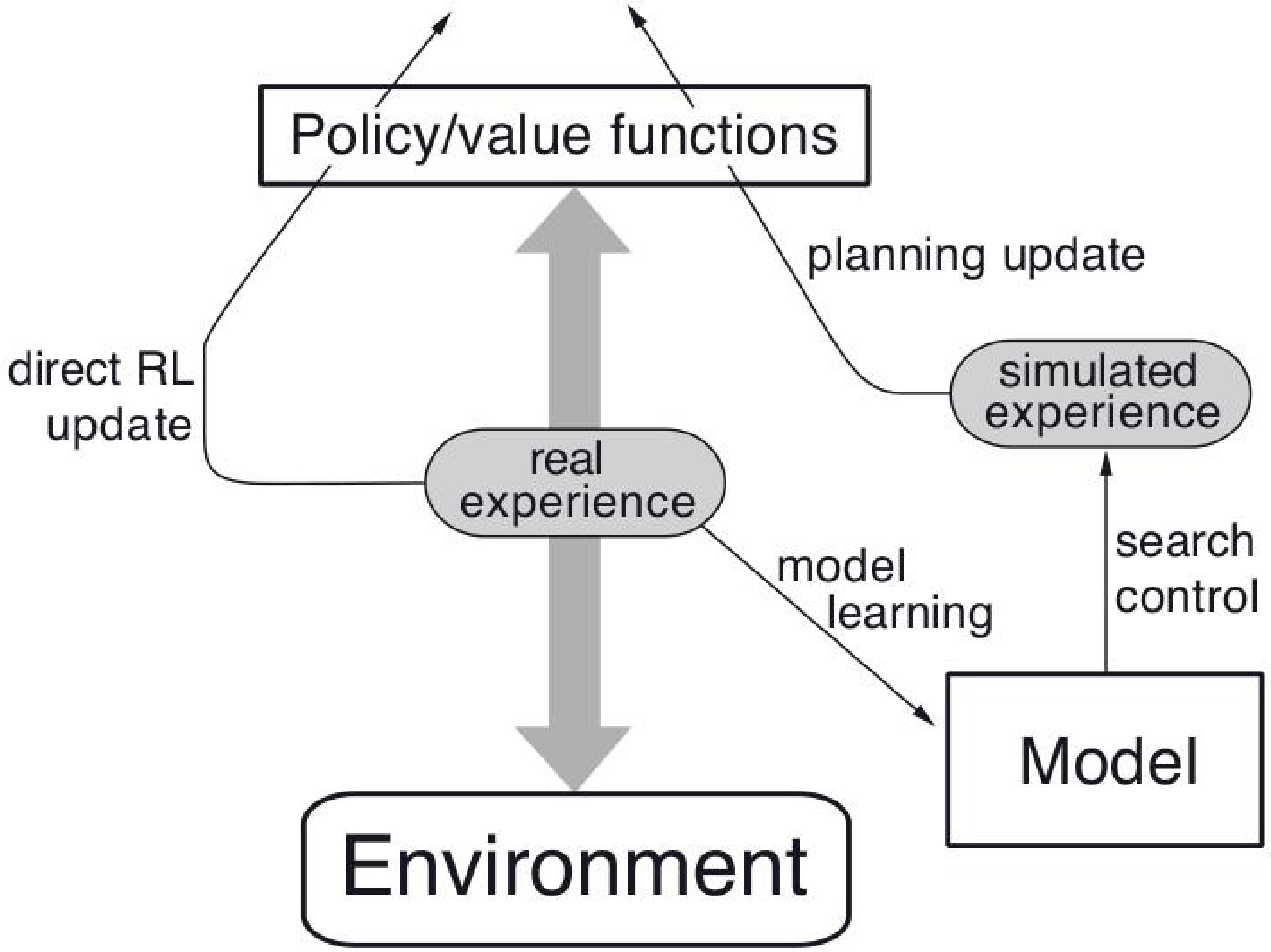
$q(s, a, \vec{w})$?





Going beyond - Dyna-Q integration planning, acting, learning





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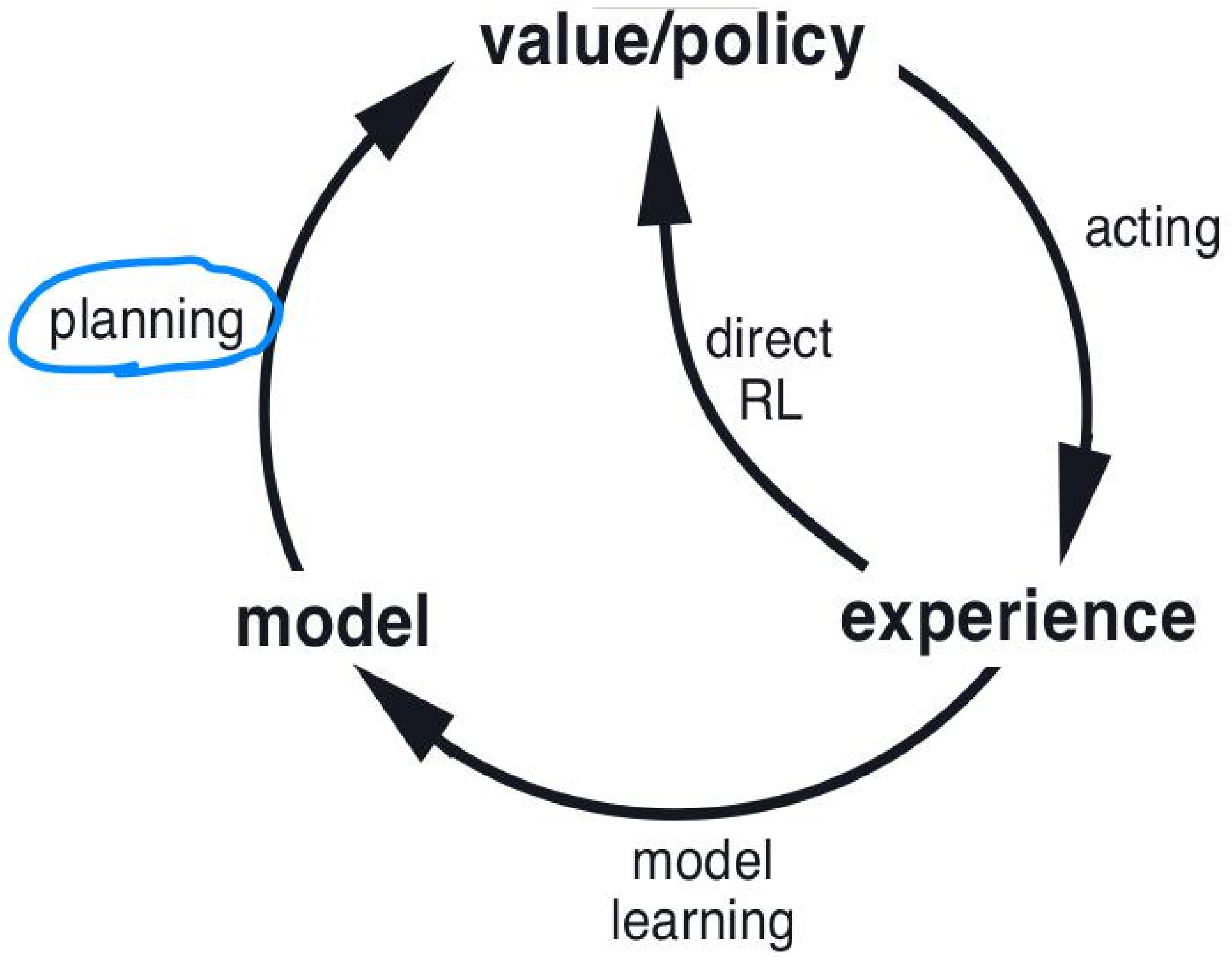
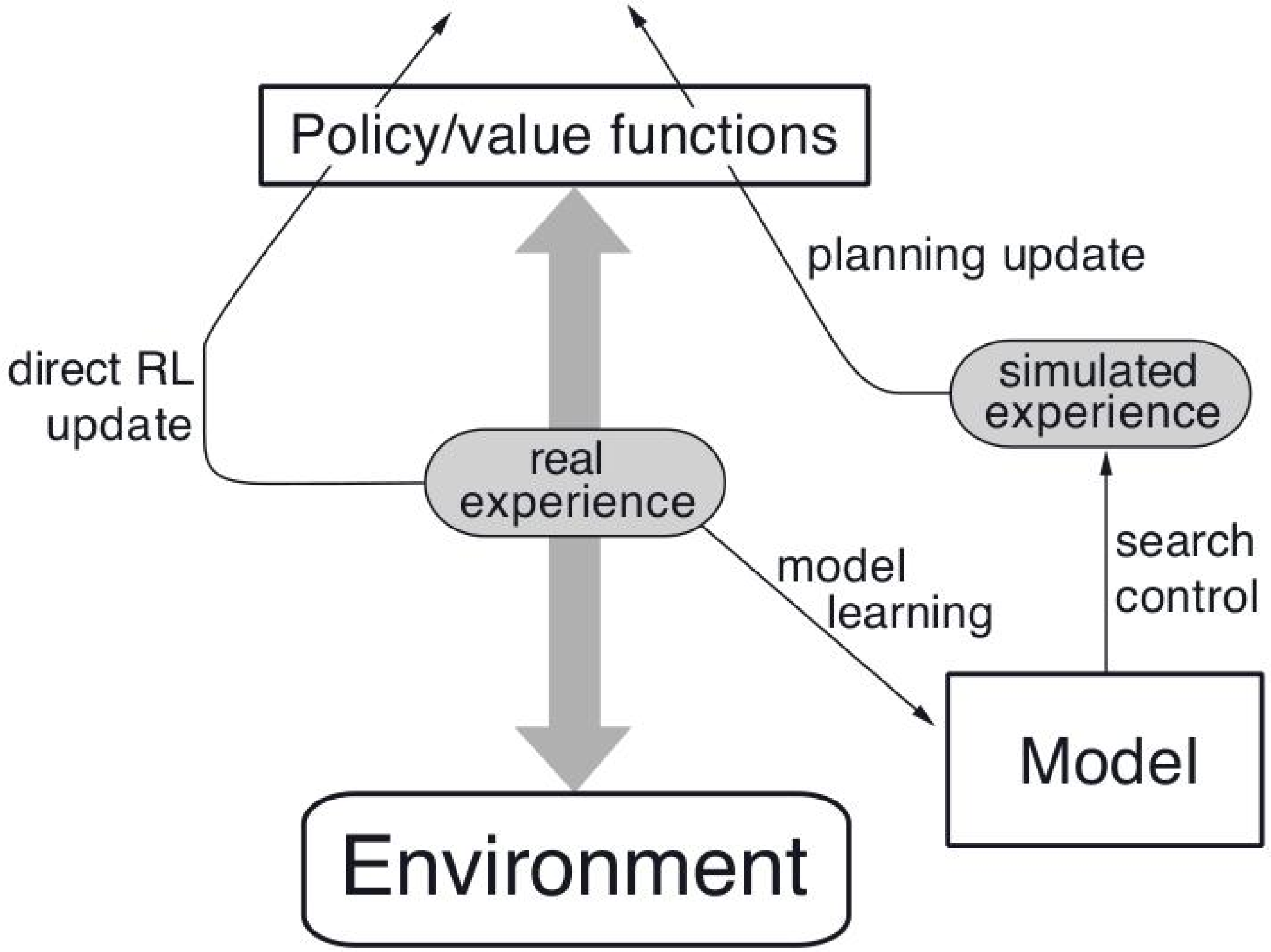
[2]Schemes from [2]

Going beyond - Dyna-Q integration planning, acting, learning





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[2]Schemes from [2]

References

Further reading: Chapter 21 of [1]. More detailed discussion in [2] Chapters 6 and 9. You can read about strategies for exploratory moves at various places, Tensor Flow related[3]. More RL URLs at the course pages[4].

[1] Stuart Russell and Peter Norvig.
Artificial Intelligence: A Modern Approach.
Prentice Hall, 3rd edition, 2010.
http://aima.cs.berkeley.edu/.

[2] Richard S. Sutton and Andrew G. Barto.
Reinforcement Learning; an Introduction.
MIT Press, 2nd edition, 2018.
http://www.incompleteideas.net/book/bookdraft2018jan1.pdf.

[3] https://medium.com/emergent-future/simple-reinforcement-learning-with-tensorflow-part-7-action-selection-strategies-for-exploration-d3a97b7cceaf

[4] https://cw.fel.cvut.cz/wiki/courses/b3b33kui/cviceni/program_po_tydnech/tyden_09#reinforcement_learning_plus