Linear Classifiers II

Tomáš Svoboda

Vision for Robots and Autonomous Systems, Center for Machine Perception
Department of Cybernetics
Faculty of Electrical Engineering, Czech Technical University in Prague

May 27, 2020

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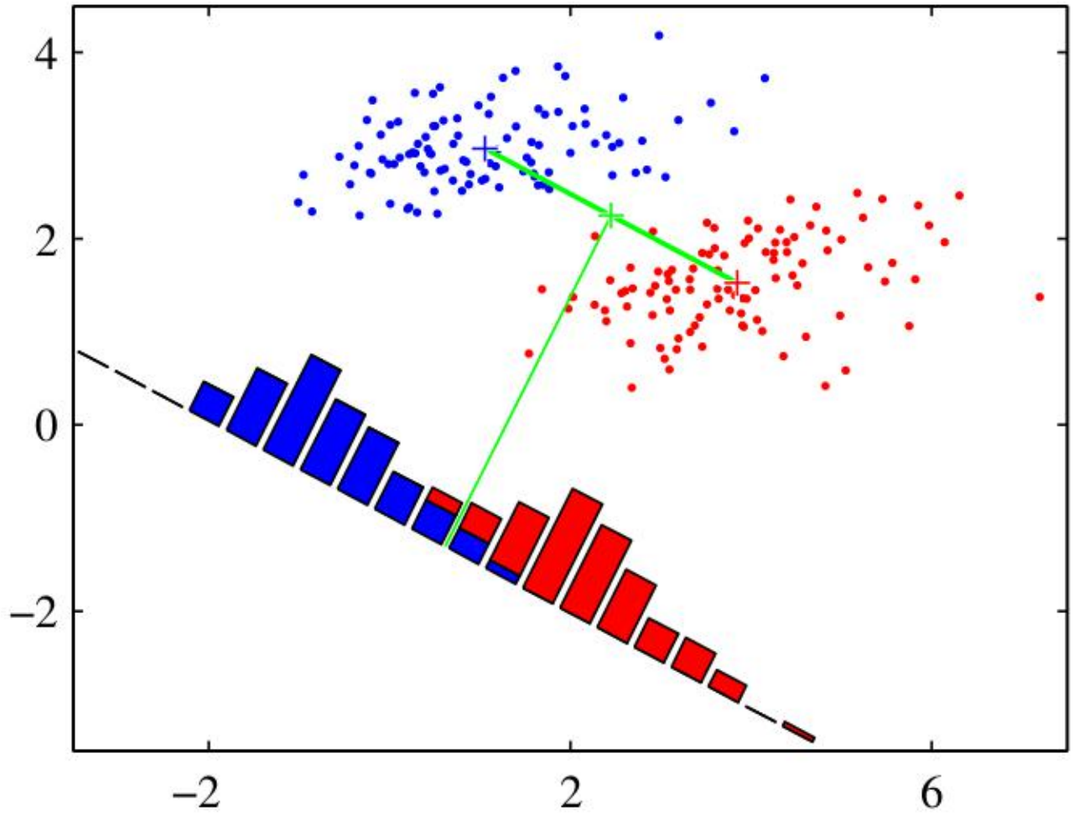
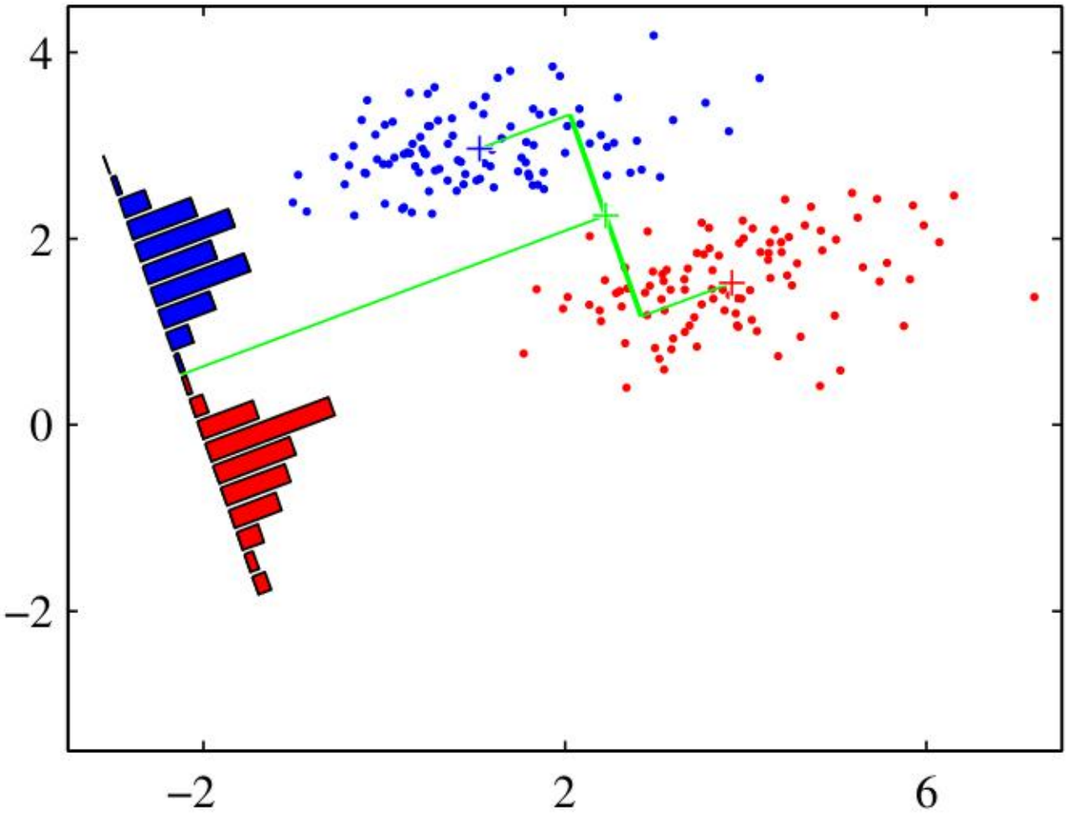
slyším TS dobře	42	100%
dostatečně	0	0%
vůbec	0	0%

Linear Classifiers - supplement lecture

- Supplement to the lecture about learning Linear Classifiers (perceptron, . . .)
- Better etalons by applying Fischer linear discriminator analysis.
- LSQ formulation of the learning task.

Fischer linear discriminant

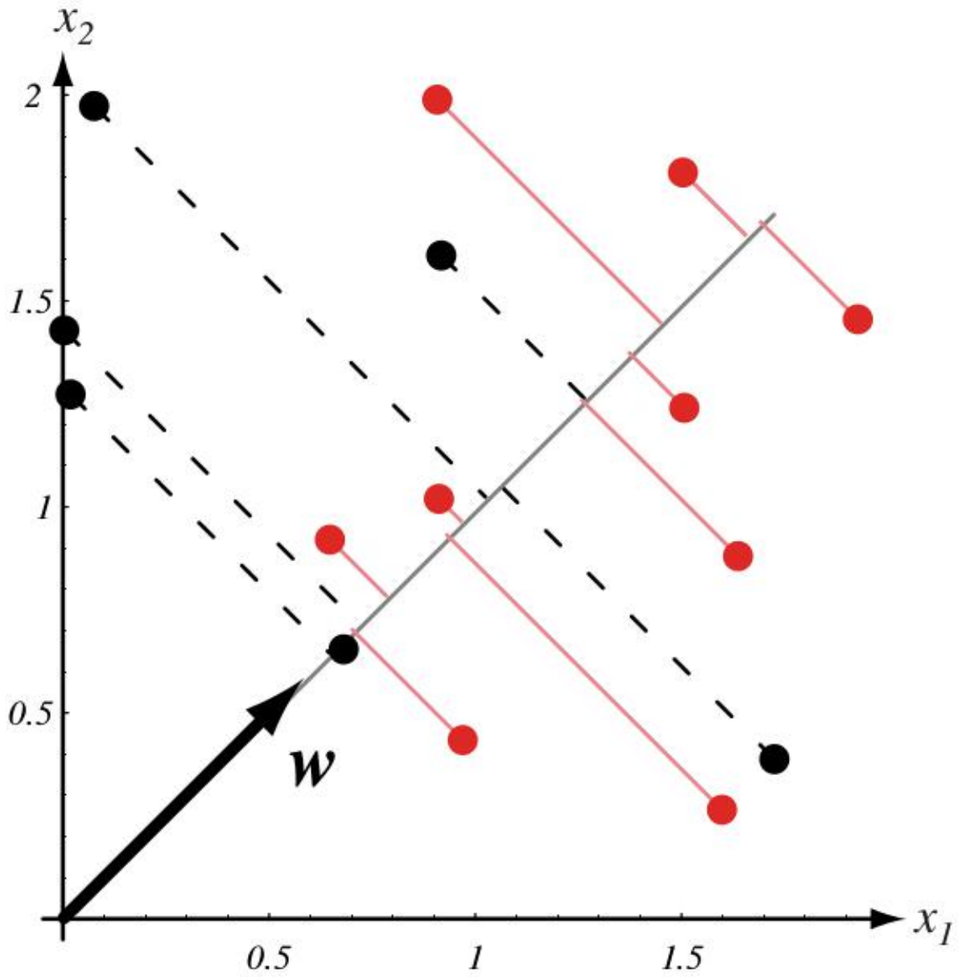




- Dimensionality reduction
- Maximize distance between means, ...
- ...and minimize within class variance. (minimize overlap)

Figures from [1]

Projections to lower dimensions $y = \mathbf{w}^\top \mathbf{x}$



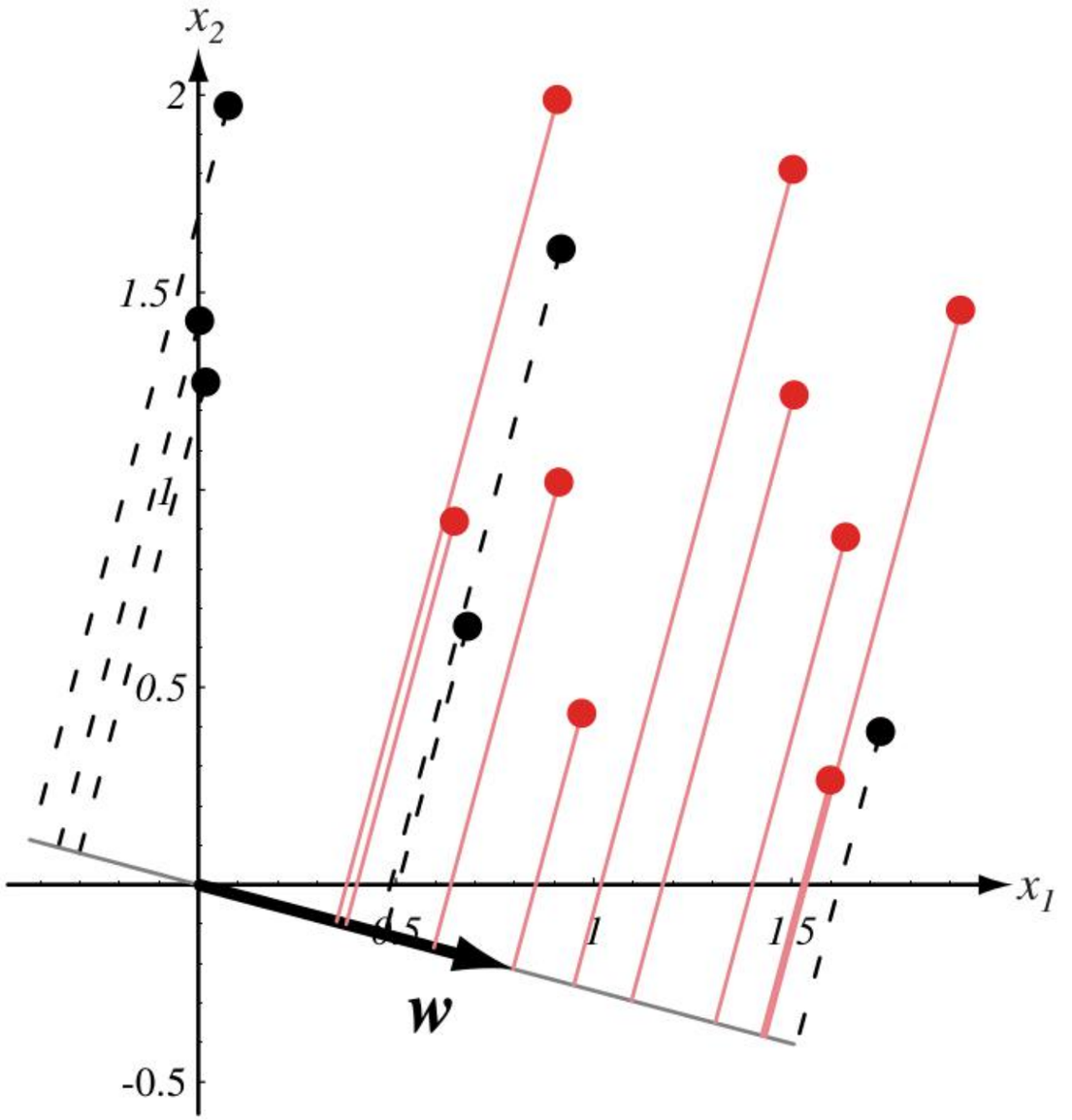
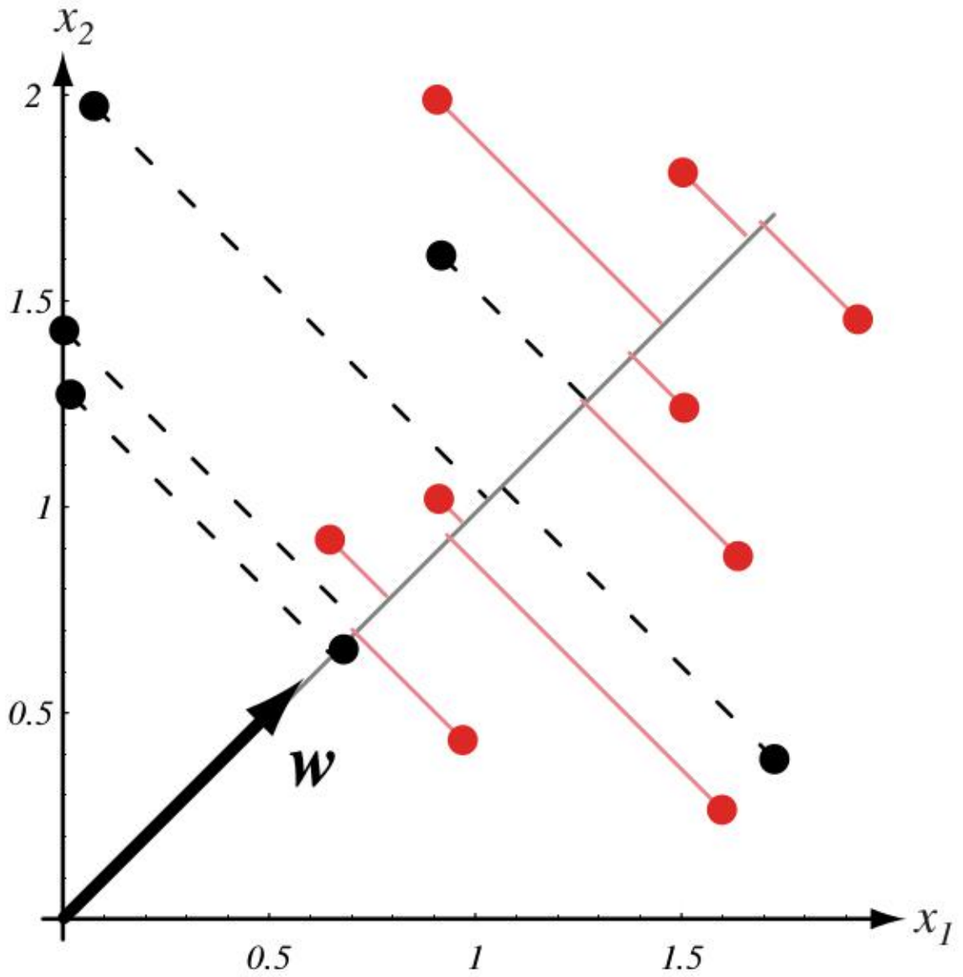
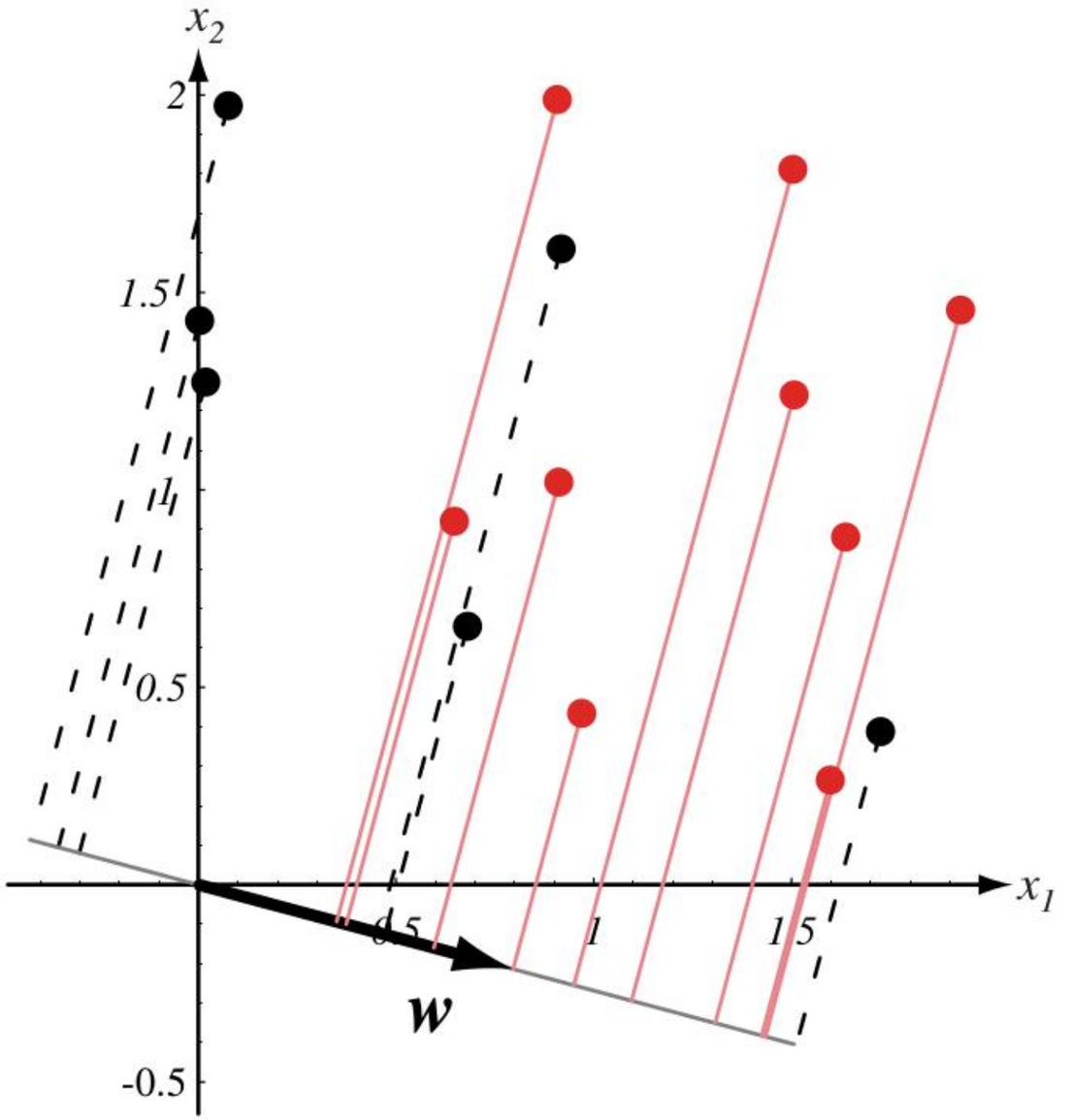


Figure from [2]

Projections to lower dimensions $y = \mathbf{w}^\top \mathbf{x}$

$\vec{x} = (x_1, x_2)^T$





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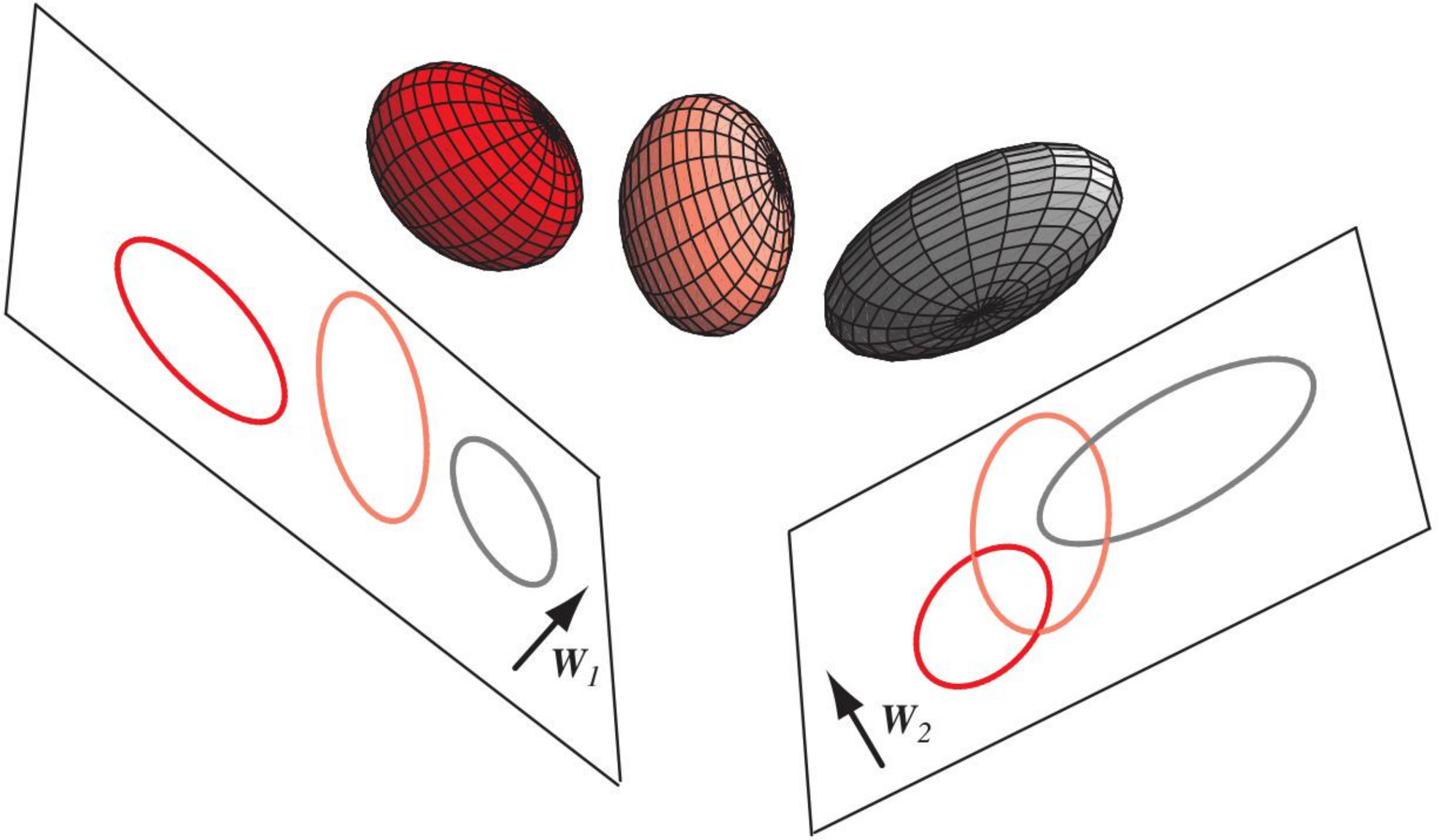

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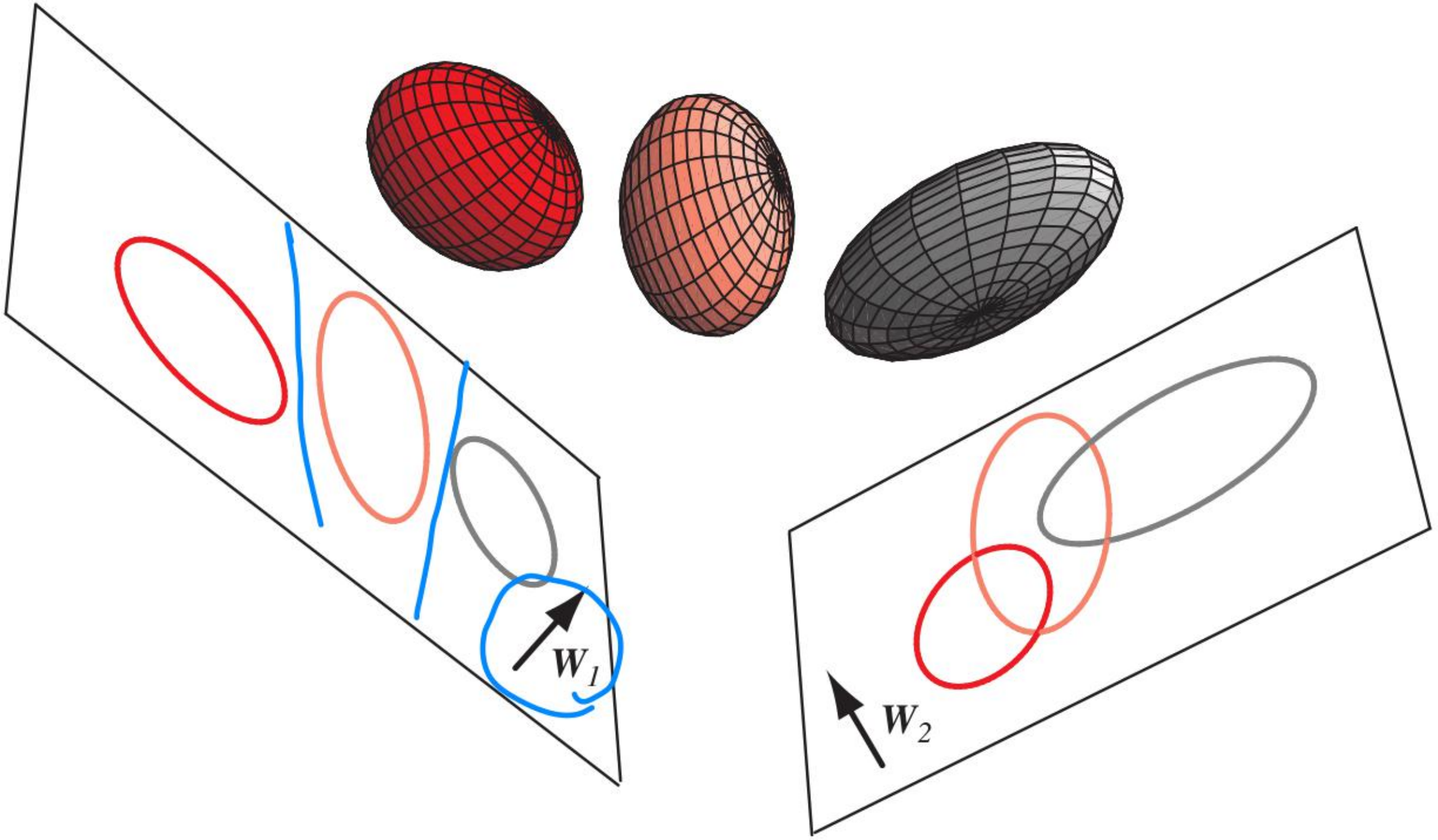
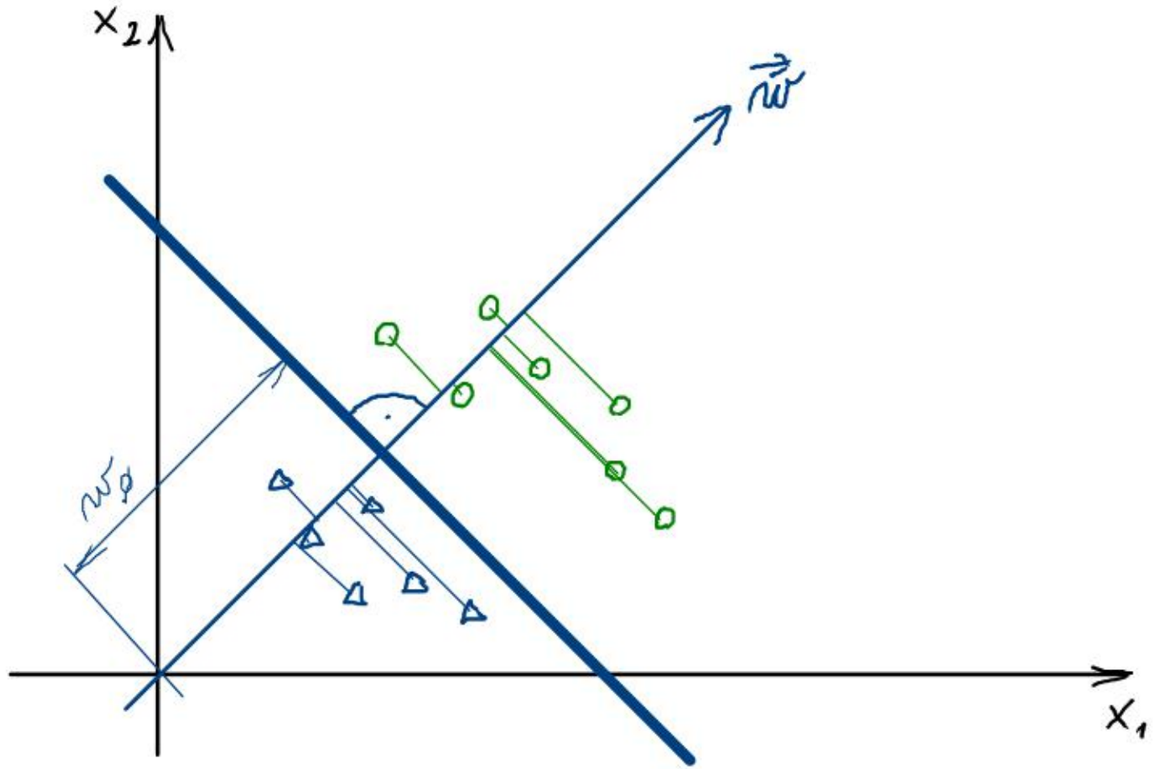
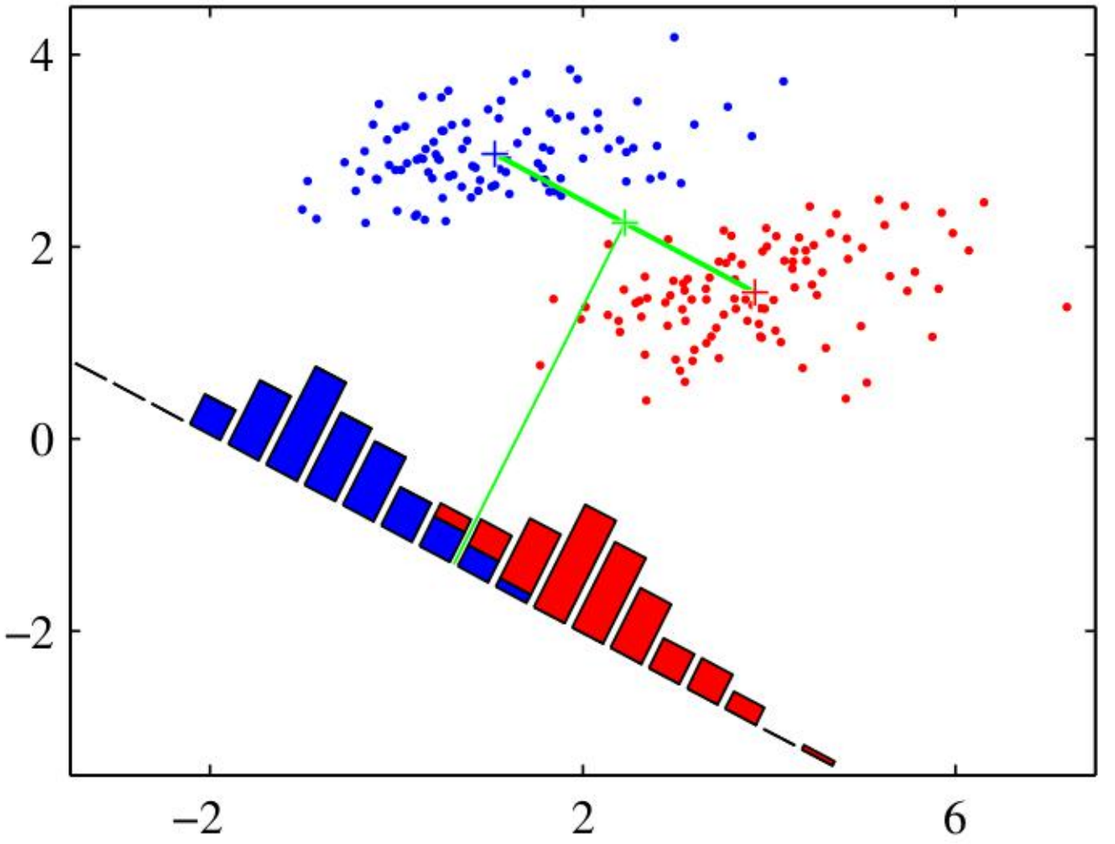


Figure from [2]

Finding the best projection $y = \mathbf{w}^\top \mathbf{x}$, $y \geq -w_0 \Rightarrow C_1$, otherwise C_2





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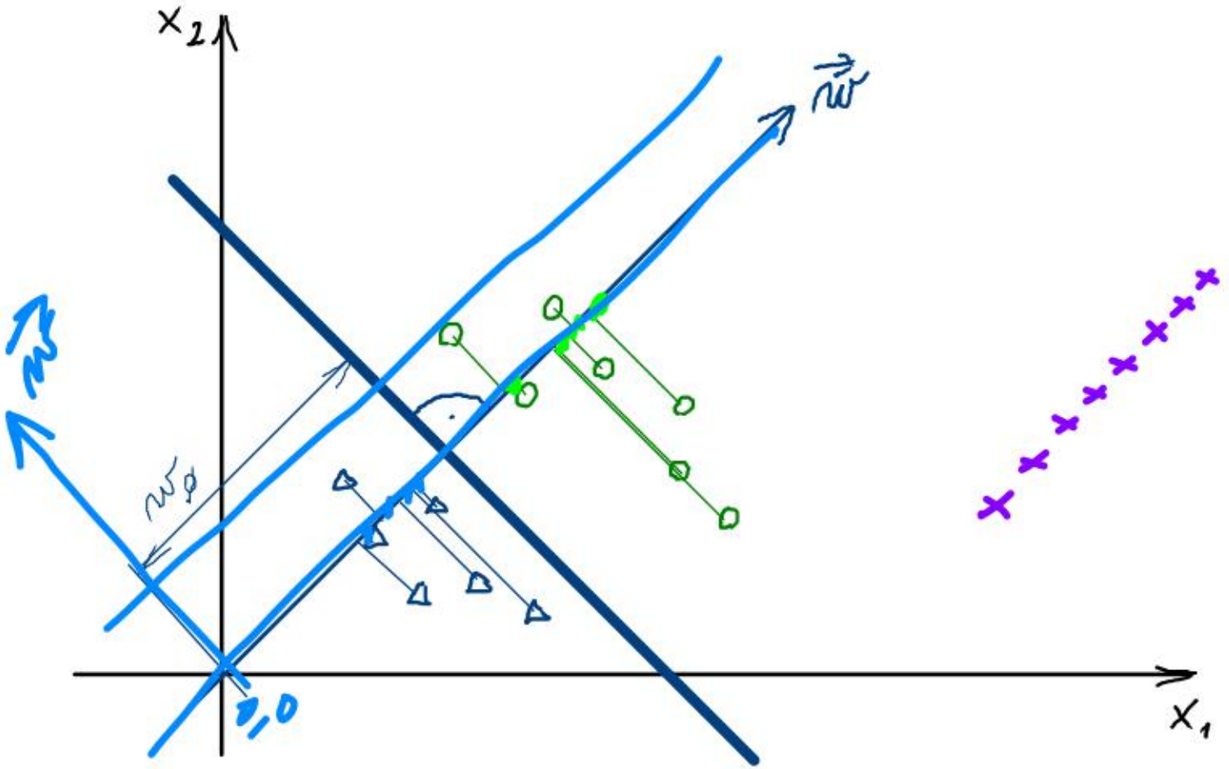
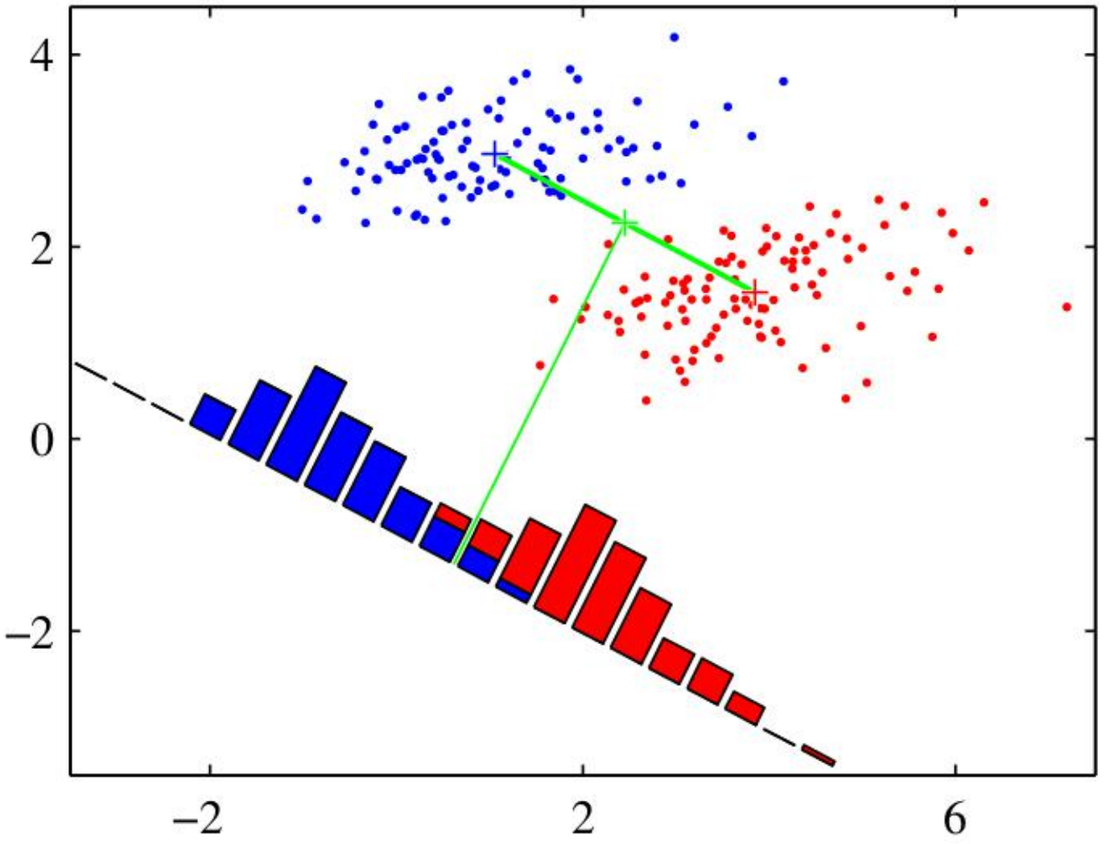




PCA

Finding the best projection $y = \mathbf{w}^\top \mathbf{x}$, $y \geq -w_0 \Rightarrow C_1$, otherwise C_2

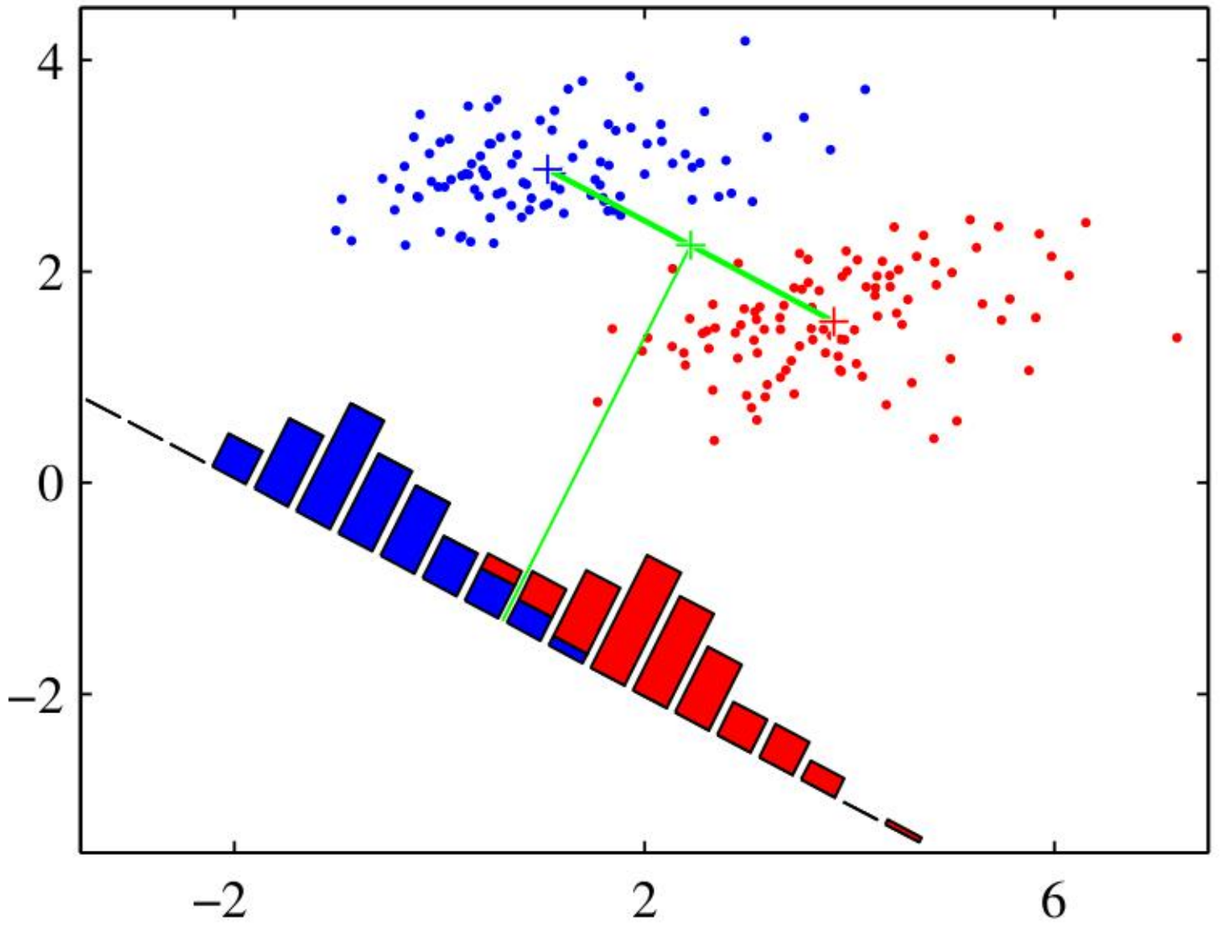
$$m_2 - m_1 = \mathbf{w}^\top (\mathbf{m}_2 - \mathbf{m}_1)$$

Within class scatter of projected samples

$$s_i^2 = \sum_{y \in C_i} (y - m_i)^2$$

Fischer criterion:

$$J(\mathbf{w}) = \frac{(m_2 - m_1)^2}{s_1^2 + s_2^2}$$



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$$s_i^2 = \sum_{y \in C_i} (y - m_i)^2$$

$$J(\mathbf{w}) = \frac{(m_2 - m_1)^2}{s_1^2 + s_2^2}$$

$$\frac{\partial J(\mathbf{w})}{\partial \mathbf{w}} = 0$$

$$\mathrm{S}_i = \sum_{x \in C_1} (\mathbf{x} - \mathbf{m}_i)(\mathbf{x} - \mathbf{m}_i)^\top$$

$$\mathrm{S}_W = \mathrm{S}_1 + \mathrm{S}_2$$

$$\mathrm{S}_B = (\mathbf{m}_2 - \mathbf{m}_1)(\mathbf{m}_2 - \mathbf{m}_1)^\top$$

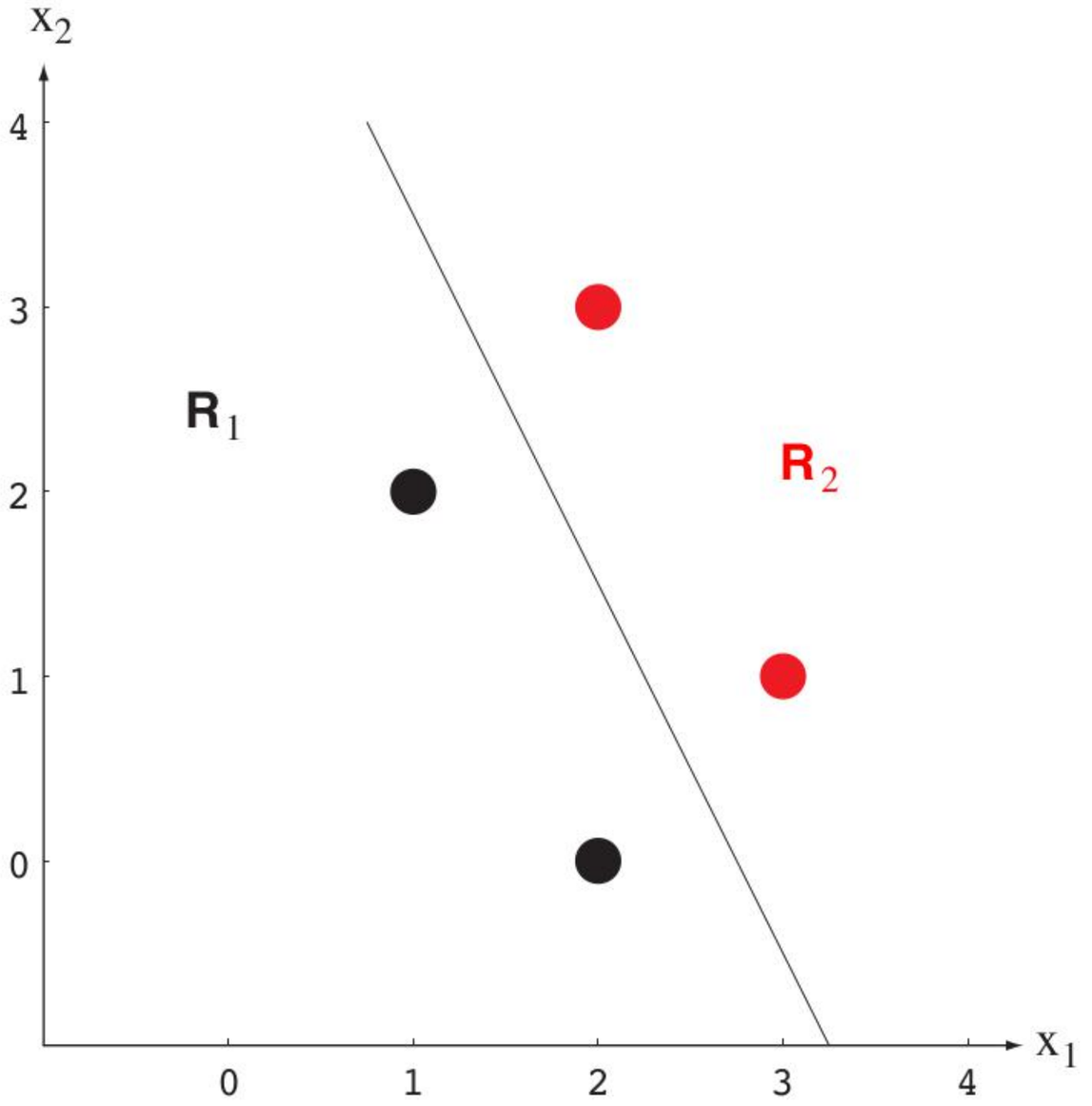
$$J(\mathbf{w}) = \frac{\mathbf{w}^\top \mathrm{S}_B \mathbf{w}}{\mathbf{w}^\top \mathrm{S}_W \mathbf{w}}$$

LSQ approach to linear classification

$$\mathbf{w} = \begin{bmatrix} w_0 \\ \mathbf{w} \end{bmatrix}$$

$$\mathtt{X}\mathbf{w} = \mathbf{b}$$

$$J(\mathbf{w}) = \|\mathtt{X}\mathbf{w} - \mathbf{b}\|^2$$



LSQ approach to linear classification

$$\begin{bmatrix} 1 & 2 & \emptyset \\ 1 & 1 & 2 \\ -1 & -3 & -1 \\ -1 & -2 & -3 \end{bmatrix} \vec{w} = \begin{bmatrix} 1 \\ 1 \\ +1 \\ -1 \end{bmatrix}$$

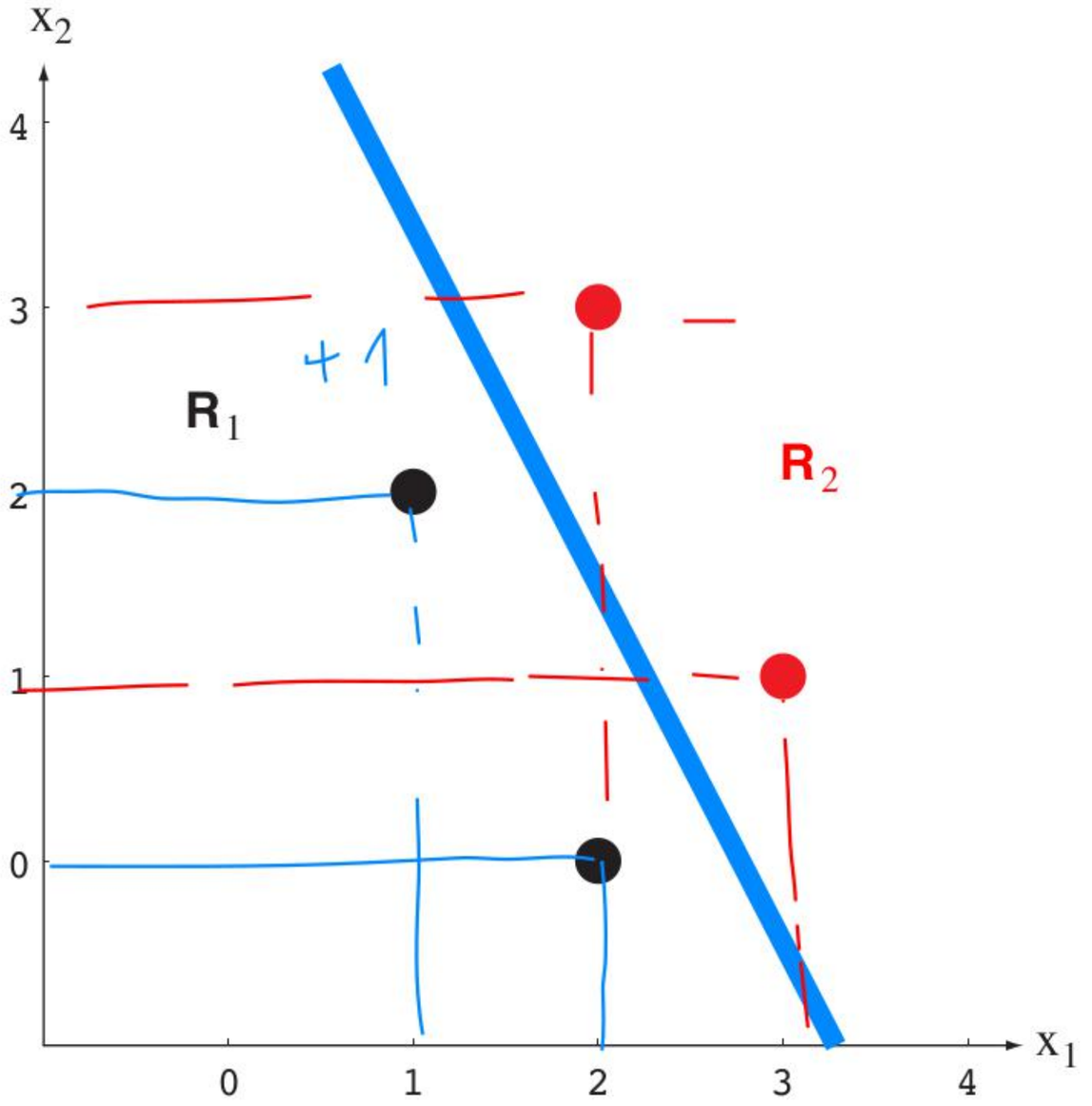
$$\tilde{\mathbf{w}} = \begin{bmatrix} w_0 \\ \mathbf{w} \end{bmatrix}$$

$$\mathrm{X}\tilde{\mathbf{w}} = \mathbf{b}$$

$$J(\mathbf{w}) = \|\mathrm{X}\mathbf{w} - \mathbf{b}\|^2$$

$$\frac{d}{d\vec{w}} \quad (X\vec{w} - \vec{b})^T (X\vec{w} - \vec{b}) = \emptyset$$

$$\vec{w} = (X^T X)^{-1} X^T \vec{b}$$



LSQ approach, better margins **b**?

$$\mathtt{X} = \begin{bmatrix} 1_1 & \mathtt{X}_1 \\ -1_2 & -\mathtt{X}_2 \end{bmatrix}$$

$$\mathbf{b} = \begin{bmatrix} \frac{n}{n_1} 1_1 \\ \frac{n}{n_2} 1_2 \end{bmatrix}$$

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References I

Further reading: Chapter 4 of [1], or chapter 3 and 5 of [2].

[1] Christopher M. Bishop.

Pattern Recognition and Machine Learning.

Springer Science+Bussiness Media, New York, NY, 2006.

PDF freely downloadable.

[2] Richard O. Duda, Peter E. Hart, and David G. Stork.

Pattern Classification.

John Wiley & Sons, 2nd edition, 2001.

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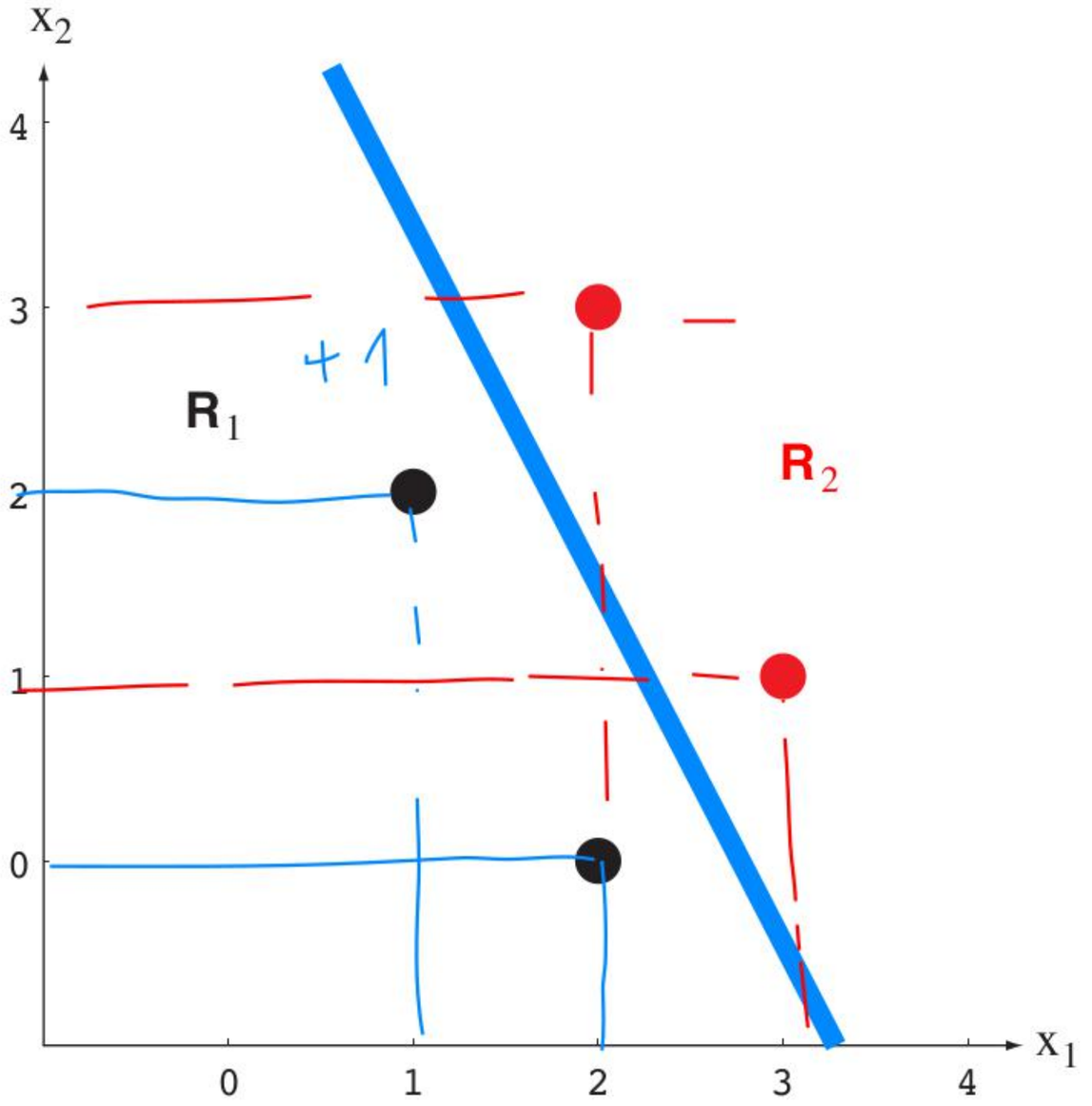

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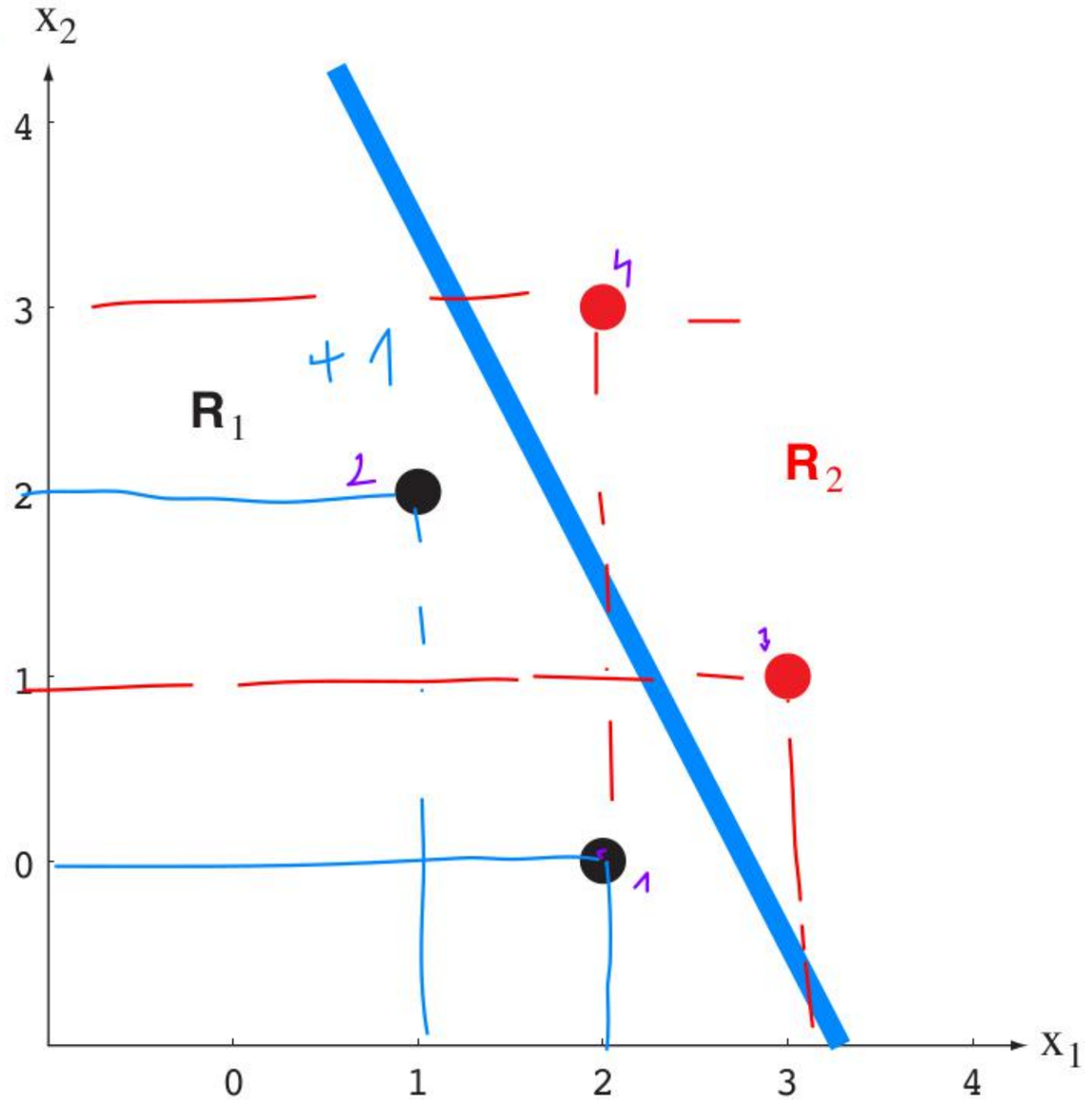


KUI closing, what next

Tomáš Svoboda
Matěj Hoffmann, Zdeněk Straka, Jana Kostlivá, Petr Švarný





Studium

- B4B33RPZ - Rozpoznávání a strojové učení
  - více o statistickém rozpoznávání (poslední 2 přednášky)
  - více matematiky,
- B3B33VIR - Vidění robotu
  - více k robotice
  - hluboké sítě, Python, PyTorch, AI-Gym …
- B0B33OPT - Optimalizace
  - minimalizace funkcí za omezení
- Magisterské studium, KyR, OI-Vision, OI-AI …

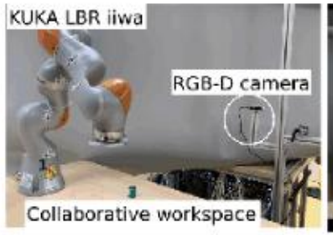
Projekty, bakalářská práce, …

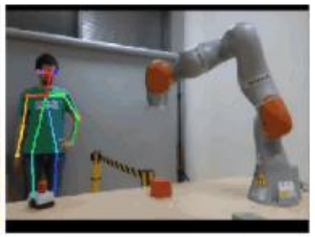
- http://www.fel.cvut.cz/en/education/semestral-projects.html
- dívejte se kolem, navštěvujte semináře, přednášky
- http://cyber.felk.cvut.cz/vras/

Student projects @ Humanoids

exploring objects by vision and touch

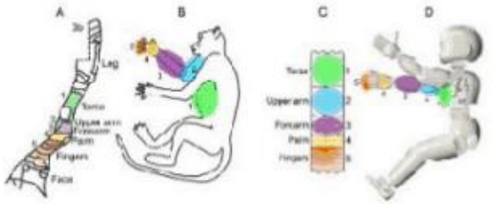
collaborative robots and human-robot interaction

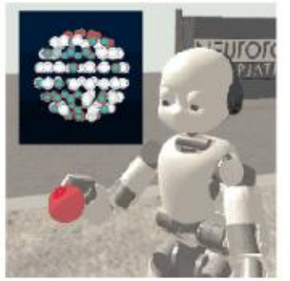




robots with artificial skin

from babies and brains to robots



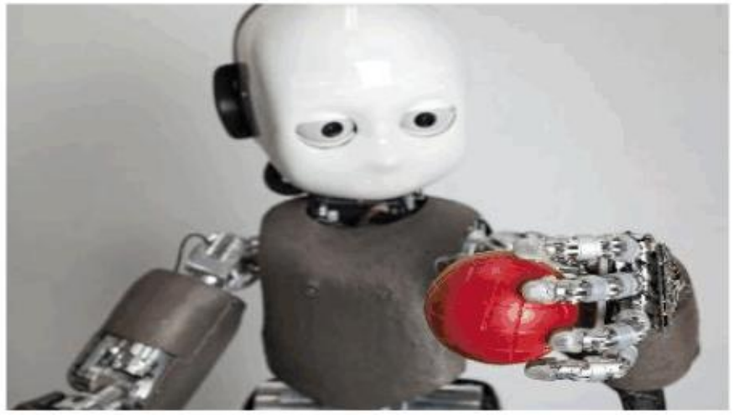


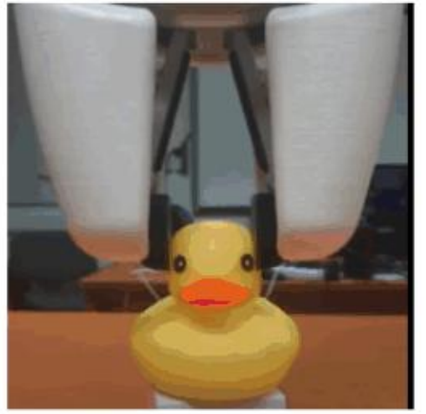
- Form: paid summer internship, bachelor project / thesis (possibly as follow-up on internship)
- Current overview: https://sites.google.com/site/matejhof/student-projects/open-and-ongoing

KUI-related project example @ Humanoids

Recognizing and exploring objects by vision and touch (soft objects)

- part of European project IPALM
- 3 robot manipulators (UR10e, KUKA LBR iiwa, Kinova Gen3) and 4 different robot grippers / hands (OnRobot RG6, Robotiq 2F-85, Barrett Hand, QB SoftHand)
- Possible task: Develop an object exploration strategy to verify the hypotheses/priors from vision about object pose, model, and properties. The actions may involve:
- manipulation (e.g., squeezing, pushing)
- visual exploration using moving camera (Intel Realsense D410 in the wrist of Kinova Gen3)





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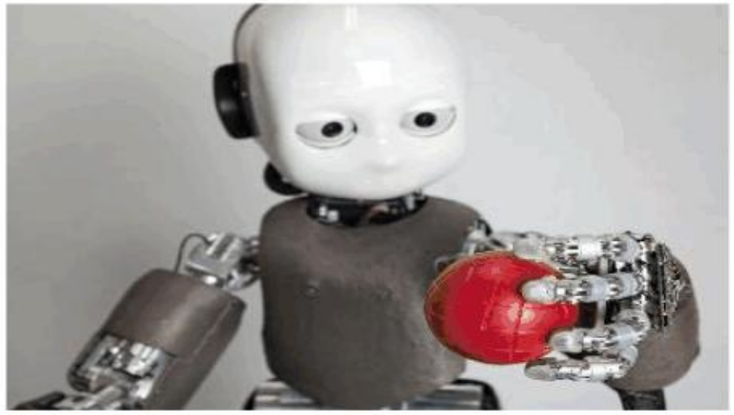

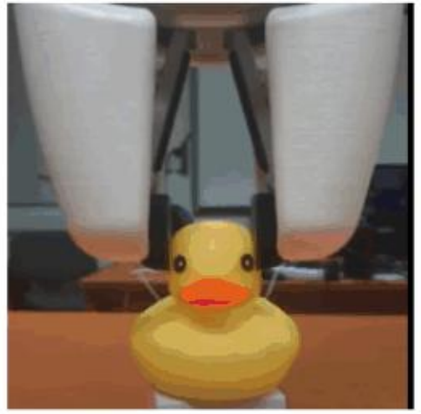

CTU-CRAS-NORLAB

@DARPA Subterranean Challenge
URBAN CIRCUIT

https://youtu.be/rTP64z52JFE

http://robotics.fel.cvut.cz/cras/darpa-subt/

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- height-map representing a **rigid** terrain …….. h_i
- weighted pointcloud representing the robot …….. $\mathbf{p}_i, m_i$

one can estimate pose $\boldsymbol{\alpha}, \mathbf{t}$ as follows:

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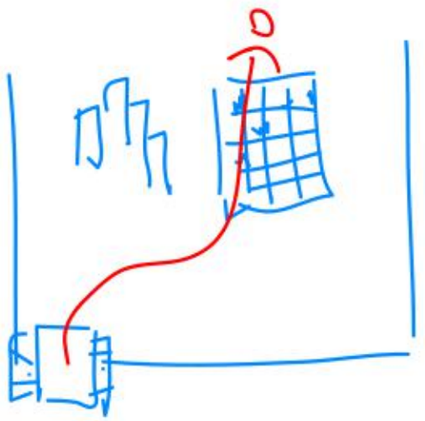

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$$\arg\min_{\boldsymbol{\alpha},\mathbf{t}} \sum_i m_i \cdot g \cdot [\mathbf{R}(\boldsymbol{\alpha}) \cdot \mathbf{p}_i + \mathbf{t}]_z$$

$$h_i - [\mathbf{R}(\boldsymbol{\alpha}) \cdot \mathbf{p}_i + \mathbf{t}]_z \leq 0 \quad \forall_i$$

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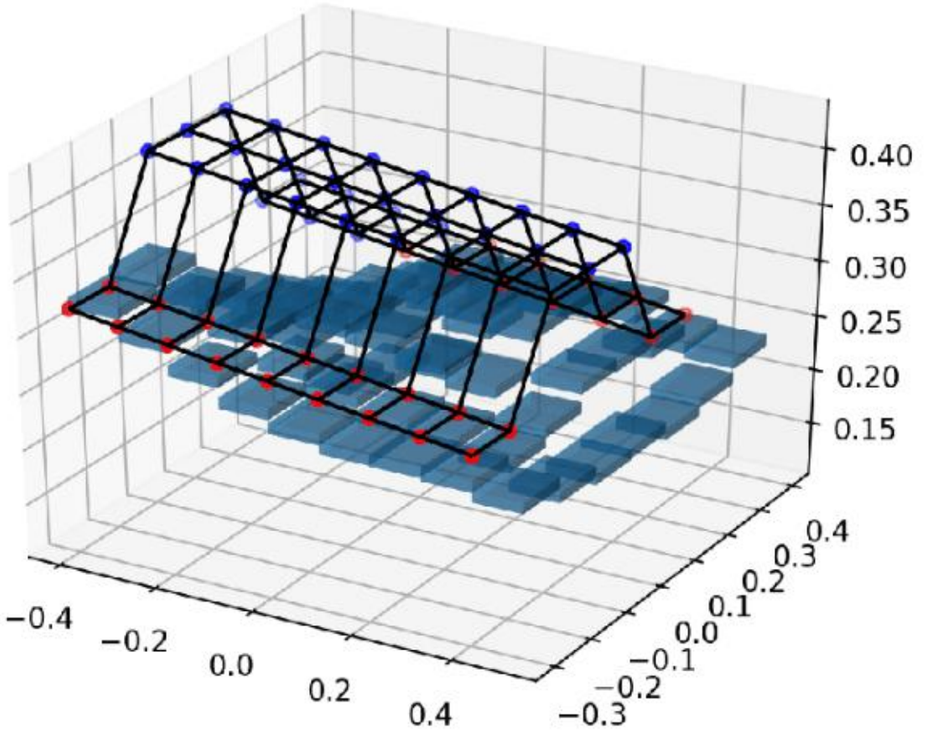

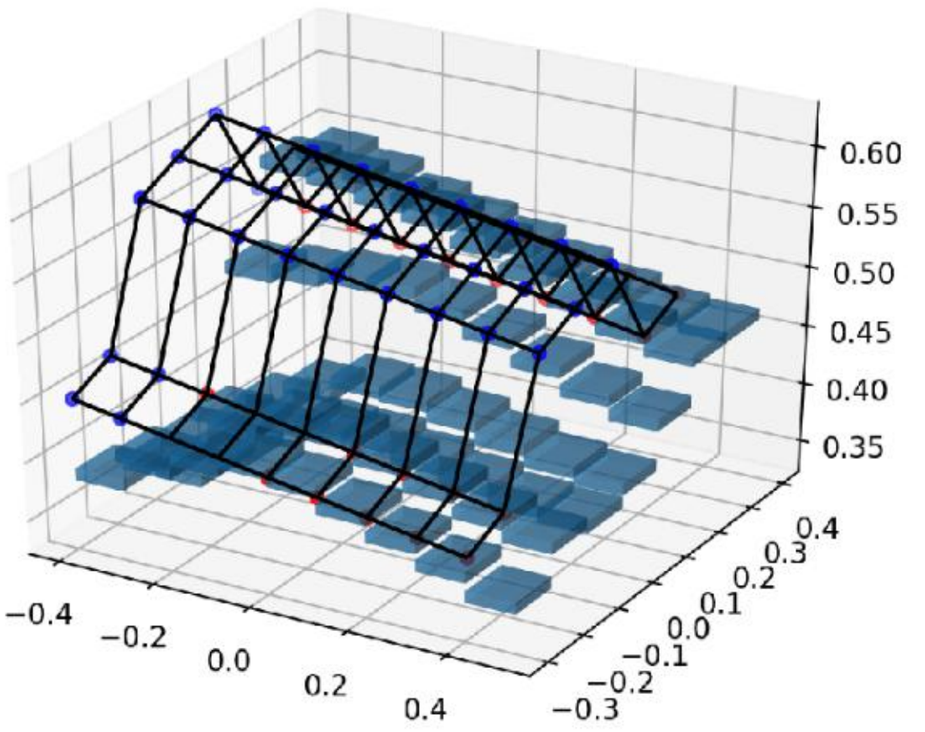

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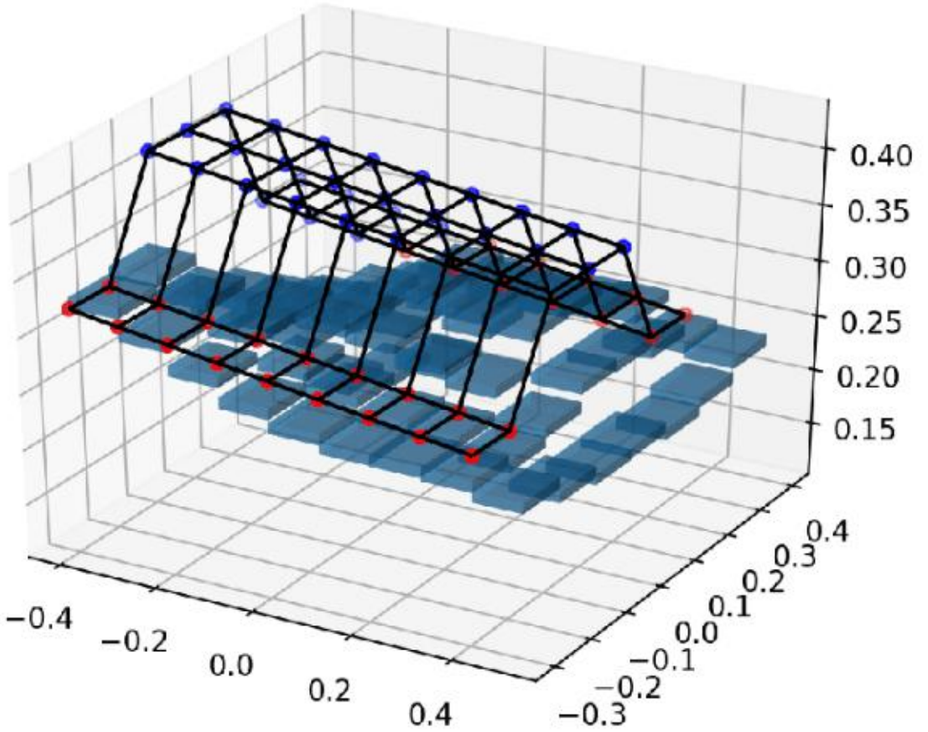
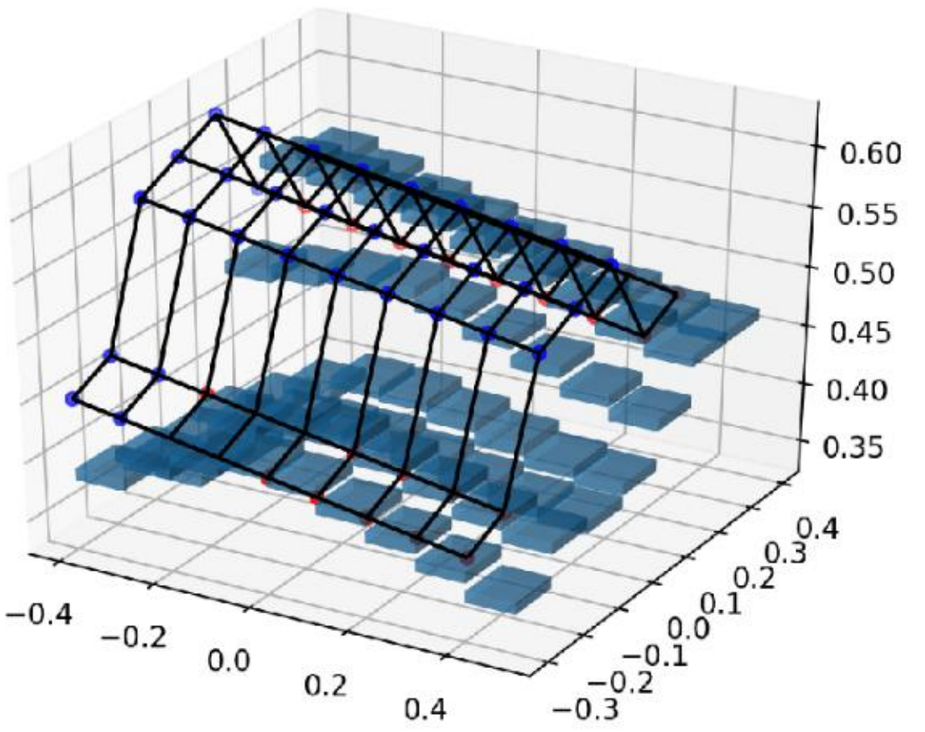
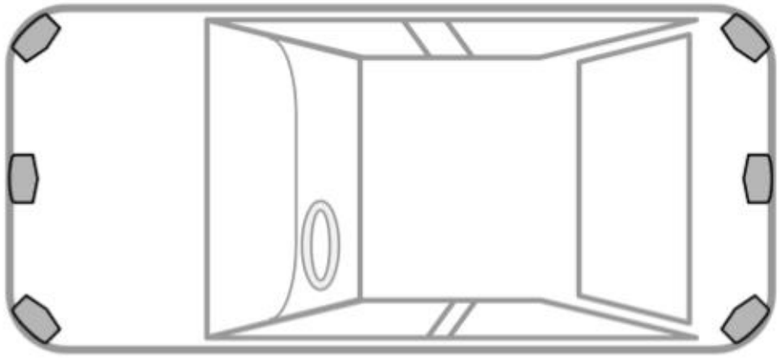
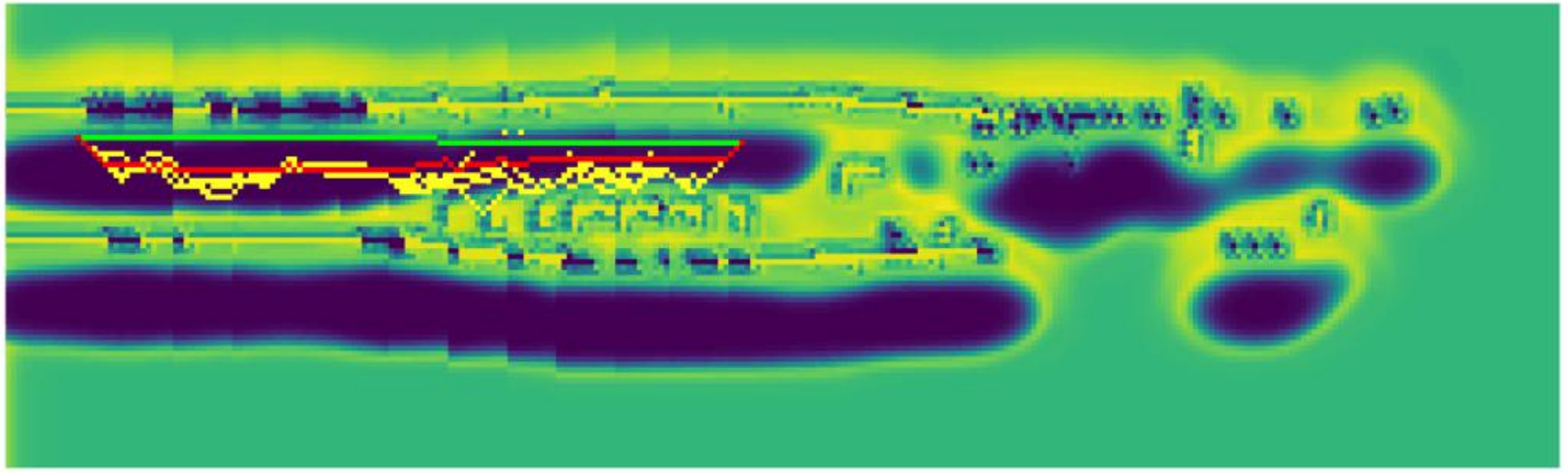
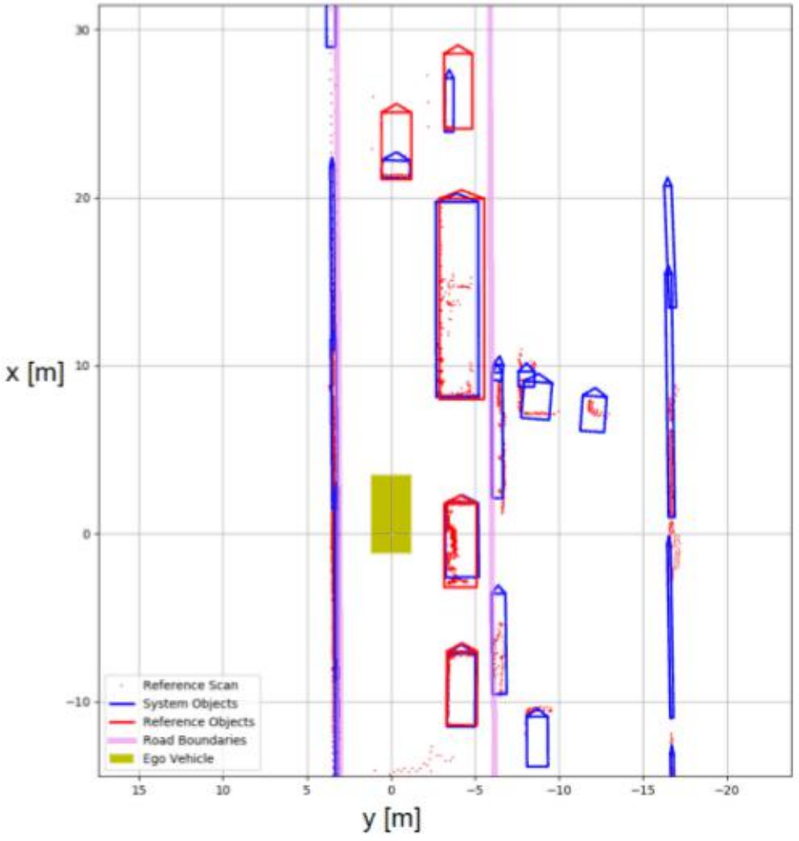


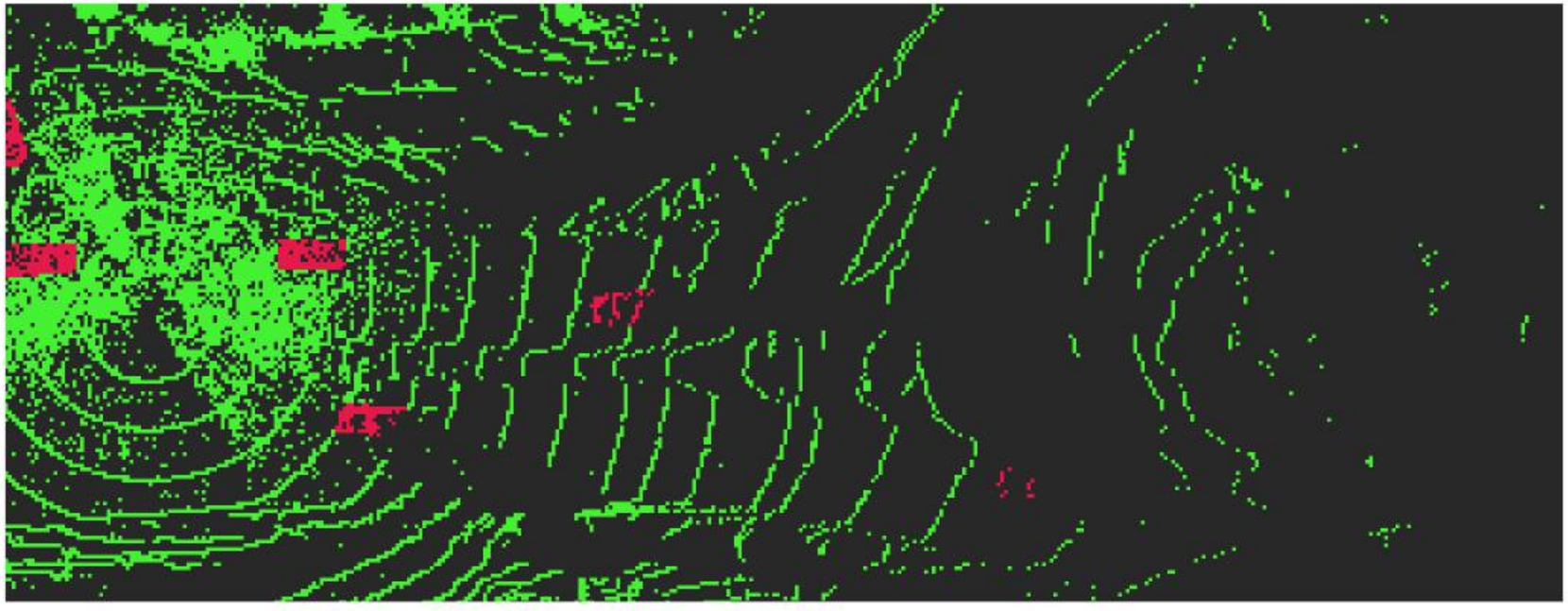


Zužitkování expertní trajektorie









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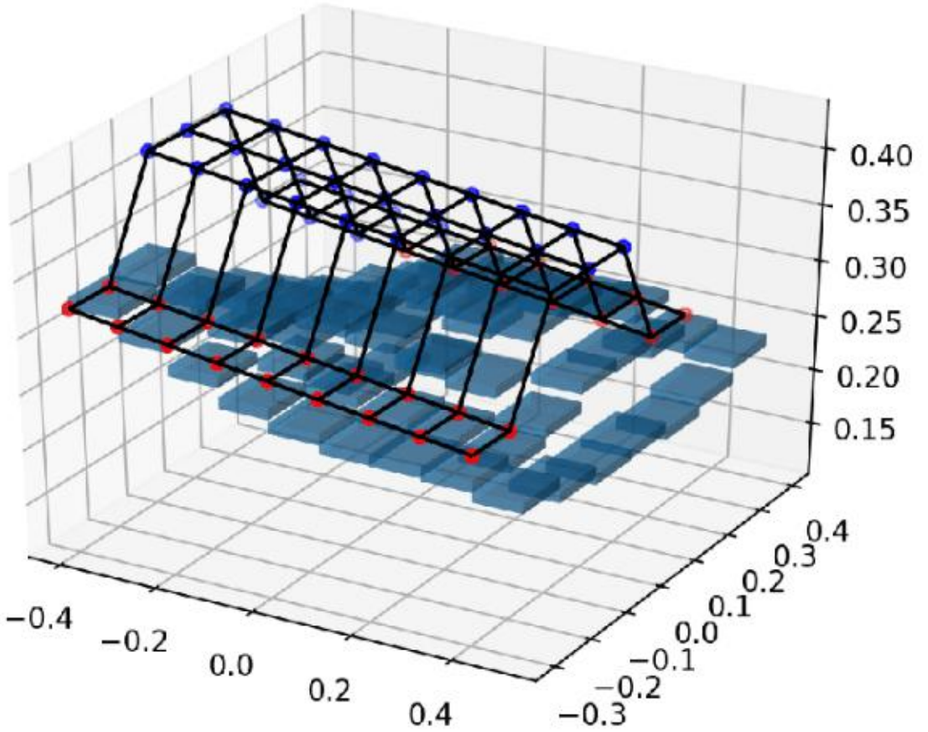

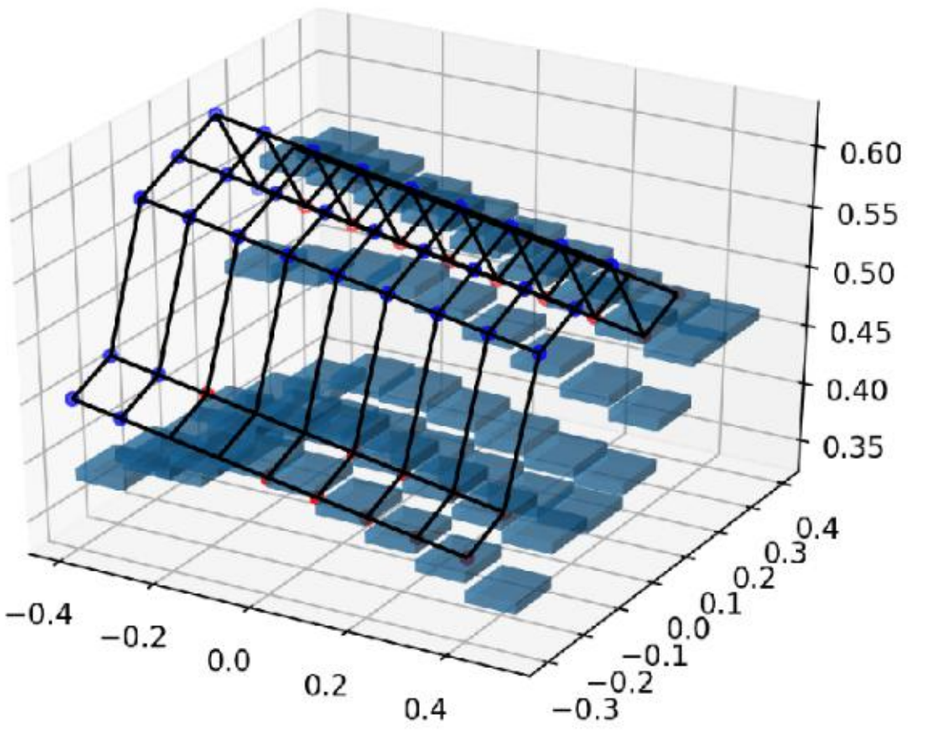

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- weighted pointcloud representing the robot ..... $\mathbf{p}_i, m_i$

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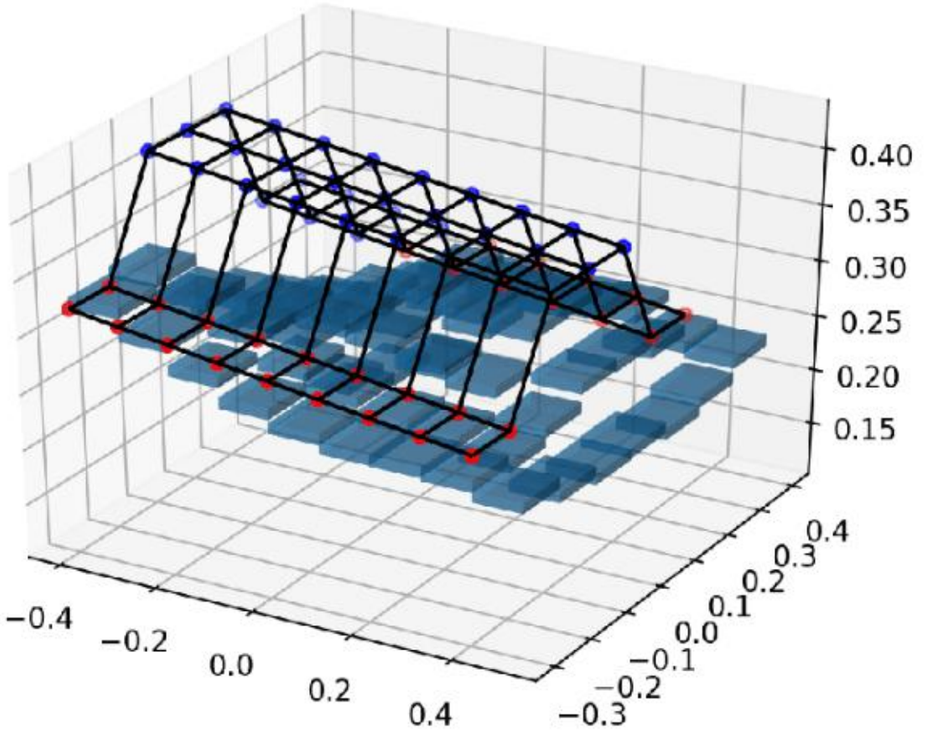

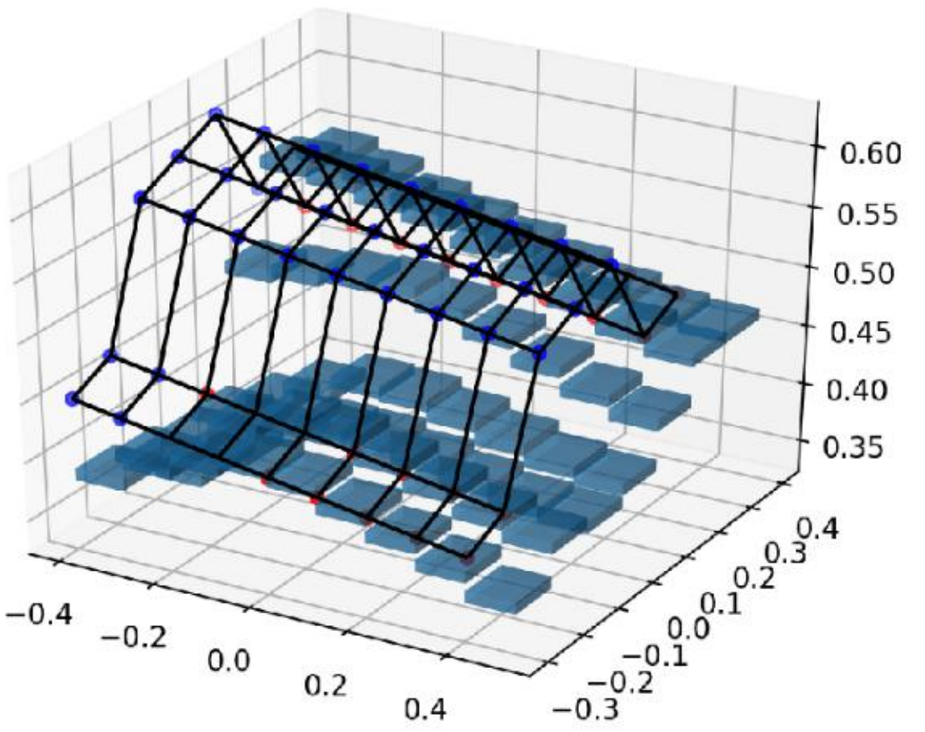

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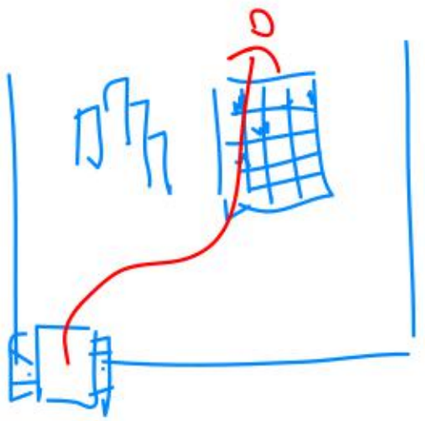

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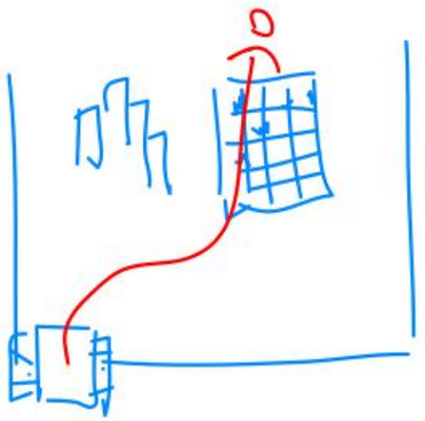

CTU-CRAS-NORLAB

@DARPA Subterranean Challenge
URBAN CIRCUIT

https://youtu.be/rTP64z52JFE

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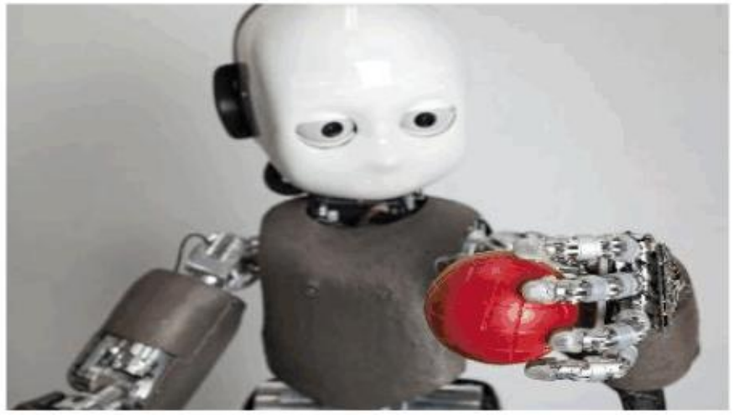
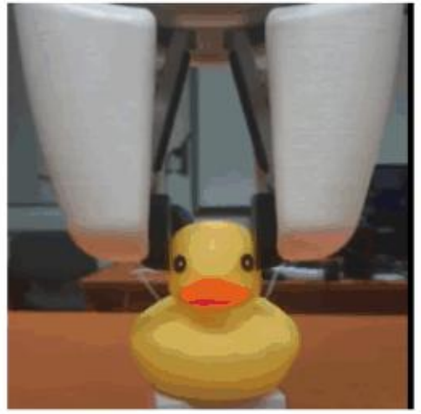
KUI-related project example @ Humanoids

Recognizing and exploring objects by vision and touch (soft objects)

- part of European project IPALM
- 3 robot manipulators (UR10e, KUKA LBR iiwa, Kinova Gen3) and 4 different robot grippers / hands (OnRobot RG6, Robotiq 2F-85, Barrett Hand, QB SoftHand)
- Possible task: Develop an object exploration strategy to verify the hypotheses/priors from vision about object pose, model, and properties. The actions may involve:
- manipulation (e.g., squeezing, pushing)
- visual exploration using moving camera (Intel Realsense D410 in the wrist of Kinova Gen3)





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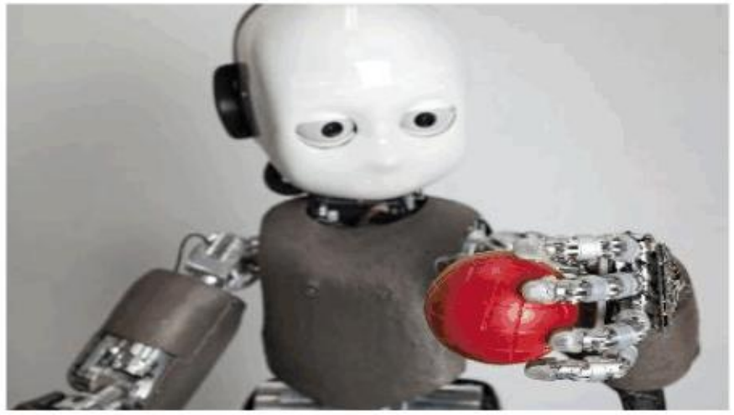
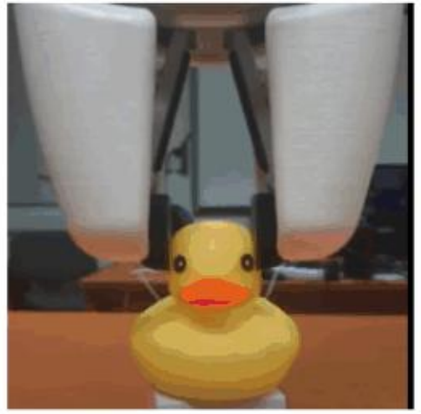
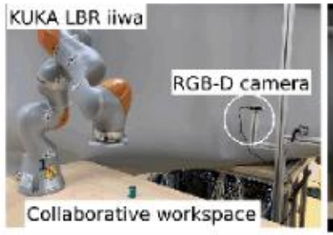


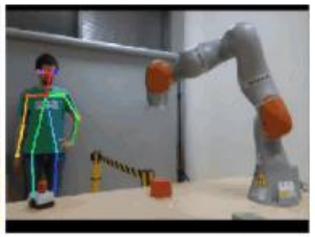


Student projects @ Humanoids

exploring objects by vision and touch

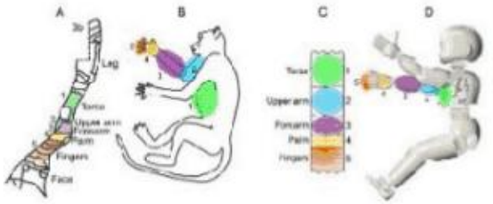
collaborative robots and human-robot interaction

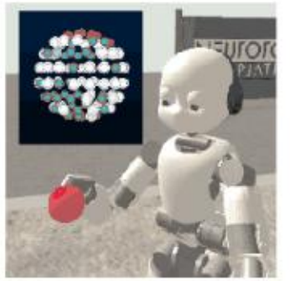




robots with artificial skin

from babies and brains to robots





- Form: paid summer internship, bachelor project / thesis (possibly as follow-up on internship)
- Current overview: https://sites.google.com/site/matejhof/student-projects/open-and-ongoing

Projekty, bakalářská práce, …

- http://www.fel.cvut.cz/en/education/semestral-projects.html
- dívejte se kolem, navštěvujte semináře, přednášky
- http://cyber.felk.cvut.cz/vras/

Studium

- B4B33RPZ - Rozpoznávání a strojové učení
  - více o statistickém rozpoznávání (poslední 2 přednášky)
  - více matematiky,
- B3B33VIR - Vidění robotu
  - více k robotice
  - hluboké sítě, Python, PyTorch, AI-Gym …
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