Reinforcement learning

Tomáš Svoboda

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Department of Cybernetics
Faculty of Electrical Engineering, Czech Technical University in Prague

April 15, 2020

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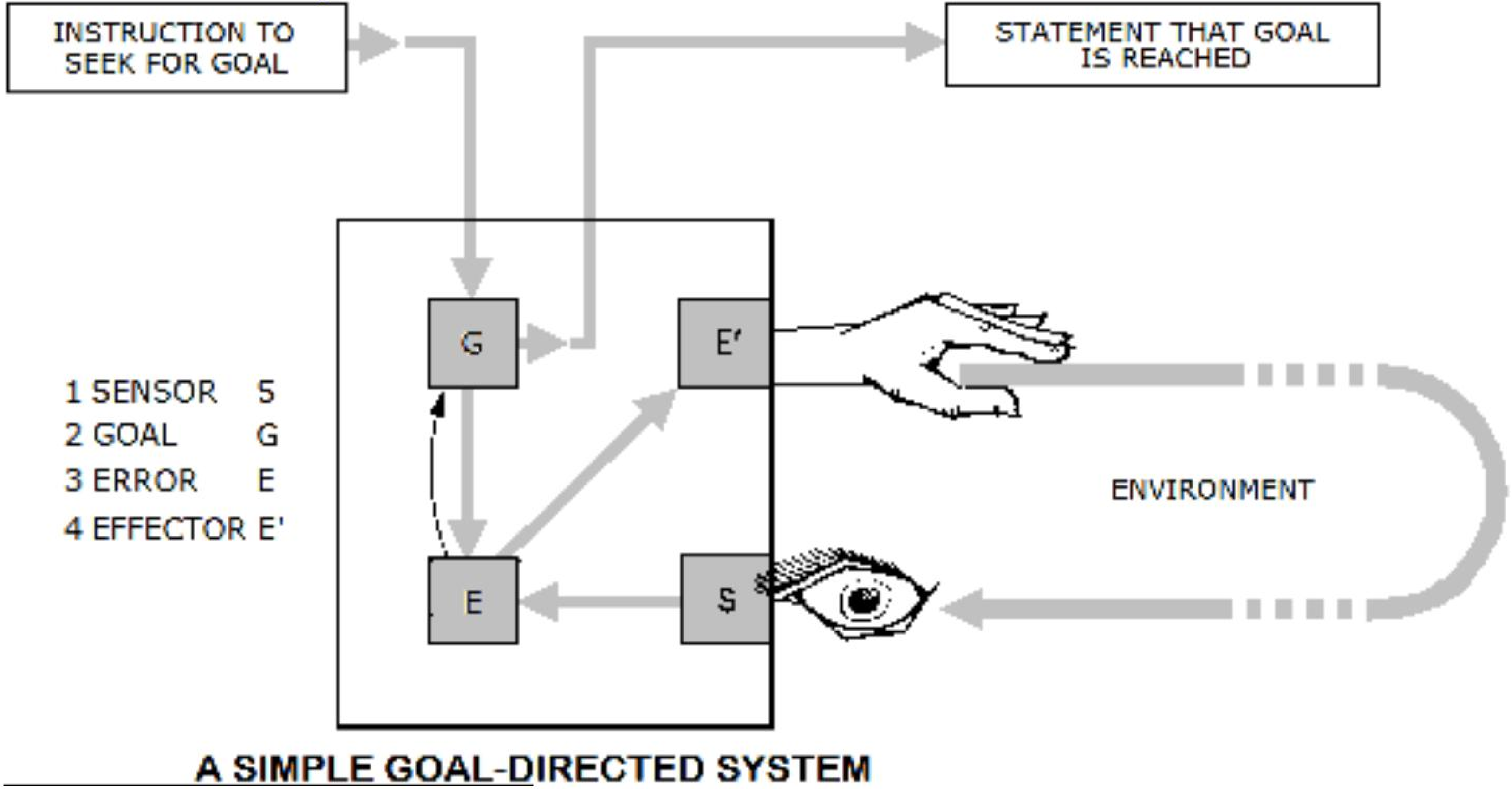
slyším velmi dobře 72%
slyším dostatečně 27%
neslyším 0%

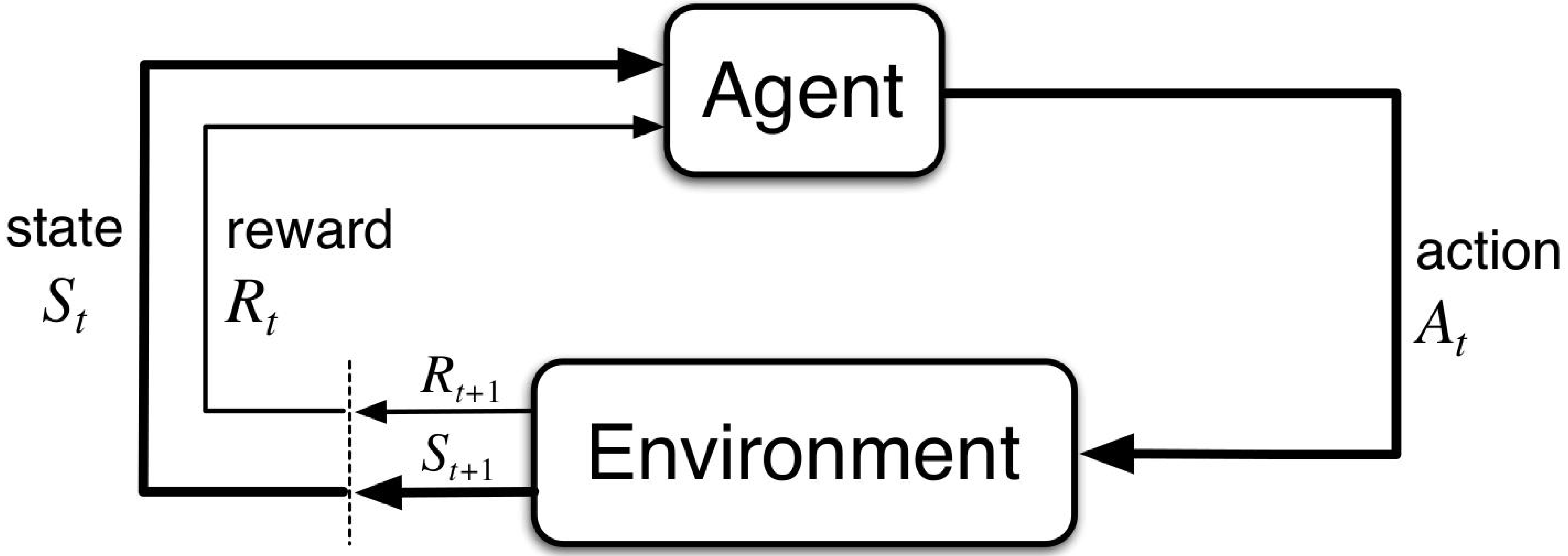
Goal-directed system



[1]

[1] Figure from http://www.cybsoc.org/gcyb.htm

Reinforcement Learning



2

- Feedback in form of Rewards
- Learn to act so as to maximize expected rewards.

[2]Scheme from [3]

Examples

Video: Learning safe policies[3]

[3] M. Pecka, V. Salansky, K. Zimmermann, T. Svoboda. Autonomous flipper control with safety constraints. In Intelligent Robots and Systems (IROS), 2016

From off-line (MDPs) to on-line (RL)

Markov decision process – MDPs. Off-line search, we know:

- A set of states $s \in \mathcal{S}$ (map)
- A set of actions per state. $a \in \mathcal{A}$
- A transition model $T(s, a, s')$ or $p(s'|s, a)$ (robot)
- A reward function $r(s, a, s')$ (map, robot)

Looking for the optimal policy $\pi(s)$. We can plan/search before the robot enters the environment.

On-line problem:

- Transition model p and reward function r not known.
- Agent/robot must act and learn from experience.

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$\pi : S \to A$

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(Transition) Model-based learning

The main idea: Do something and:

- Learn an approximate model from experiences.
- Solve as if the model was correct.

Learning MDP model:

- In s try a, observe s', count (s, a, s').
- Normalize to get and estimate of $p(s' \mid a, s)$.
- Discover (by observation) each $r(s, a, s')$ when experienced.

Solve the learned MDP.

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The main idea: Do something and:

1) ▶ Learn an approximate model from experiences.
2) ▶ Solve as if the model was correct.

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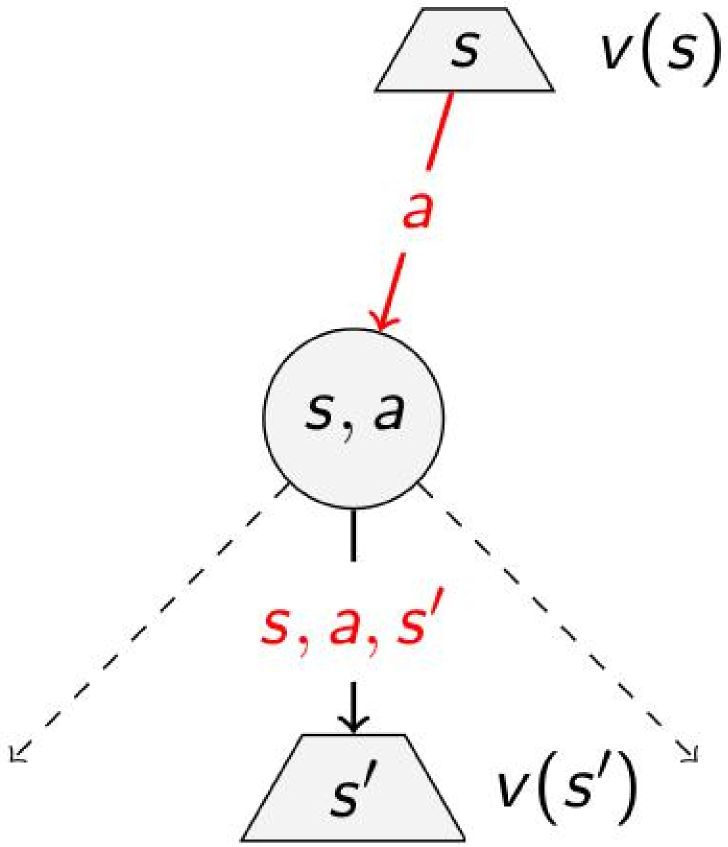
Reward function $r(s, a, s')$

- $r(s, a, s')$ - reward for taking a in s and landing in s'.
- In Grid world we assumed $r(s, a, s')$ to be the same everywhere.
- In a real world it is different (going up, down, ...)



In ai-gym `evn.step(action)` returns s', $r(s, \texttt{action}, s')$.

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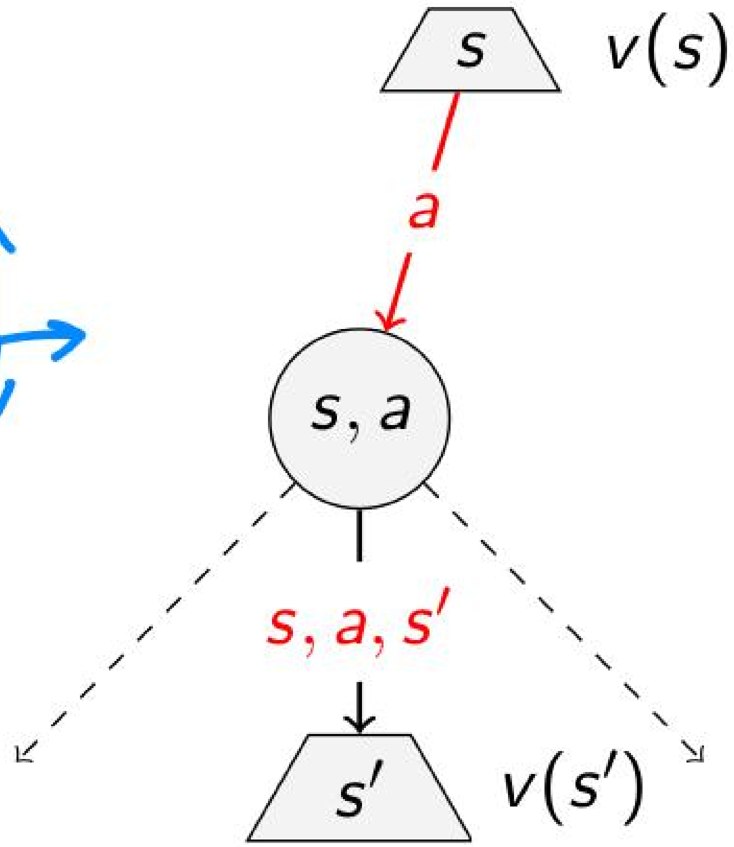

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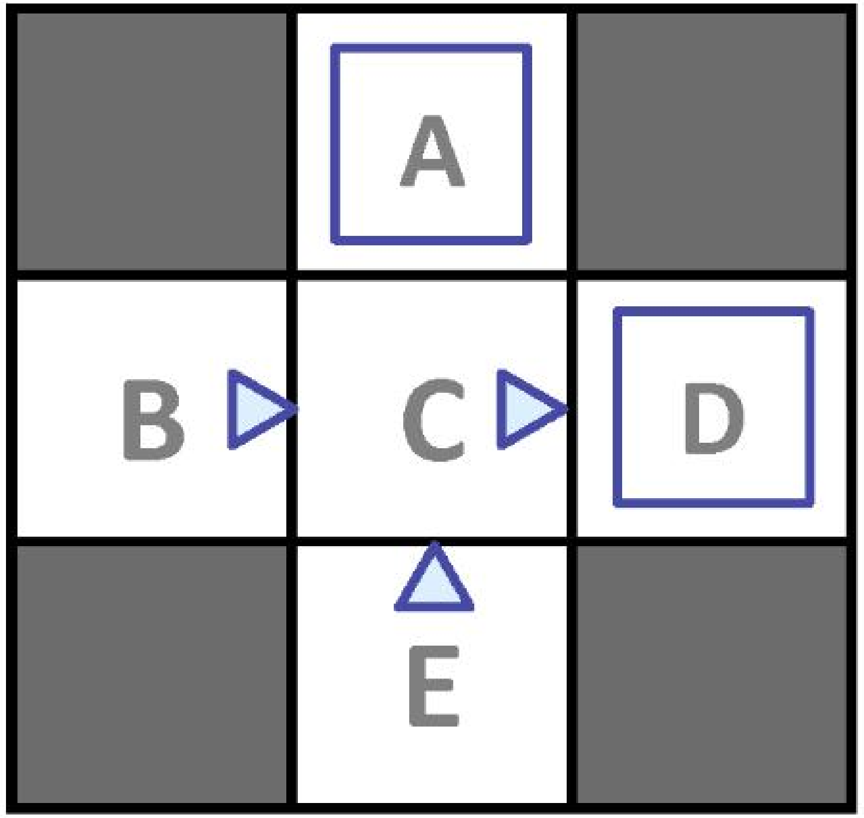
Model-based learning: Grid example

Input Policy π



Assume: $\gamma = 1$

Observed Episodes (Training)

Episode 1

B, east, C, -1
C, east, D, -1
D, exit, x, +10

Episode 2

B, east, C, -1
C, east, D, -1
D, exit, x, +10

Episode 3

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C, east, D, -1
D, exit, x, +10

Episode 4

E, north, C, -1
C, east, A, -1
A, exit, x, -10

[4]

[4] Figure from [1]

Model-based learning: Grid example

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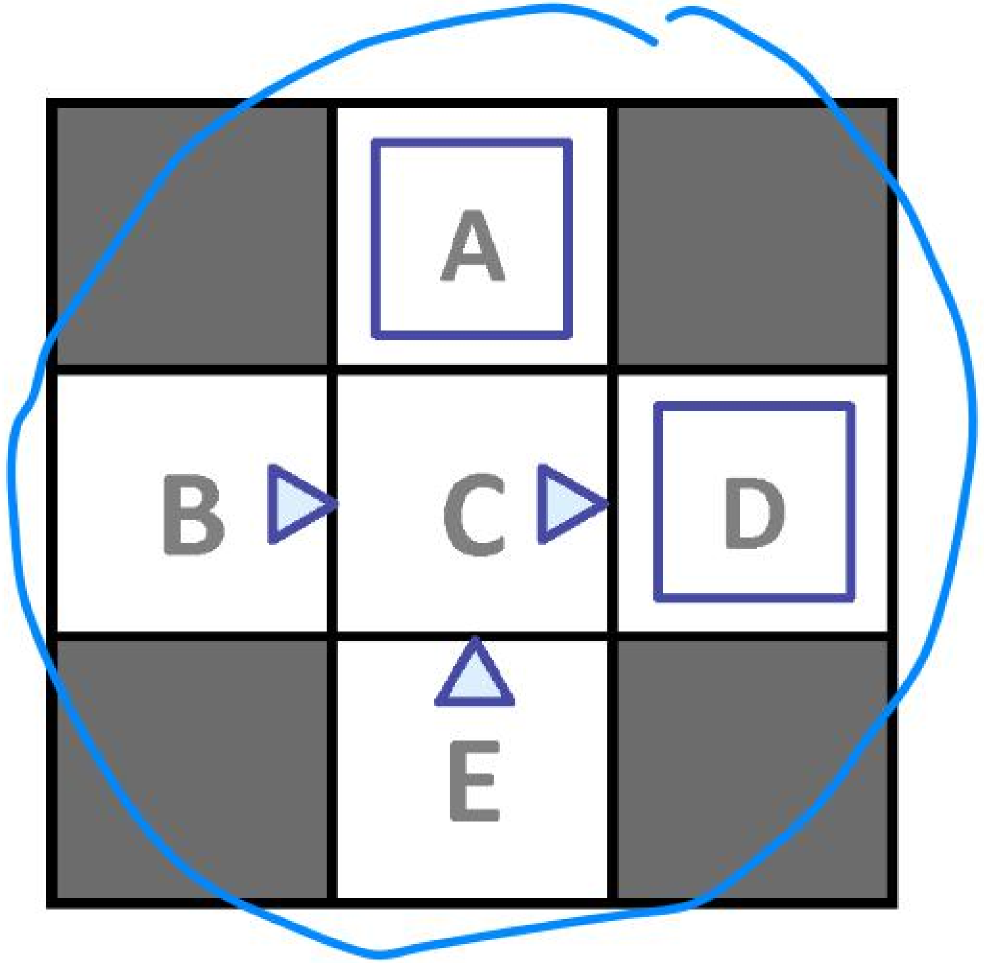

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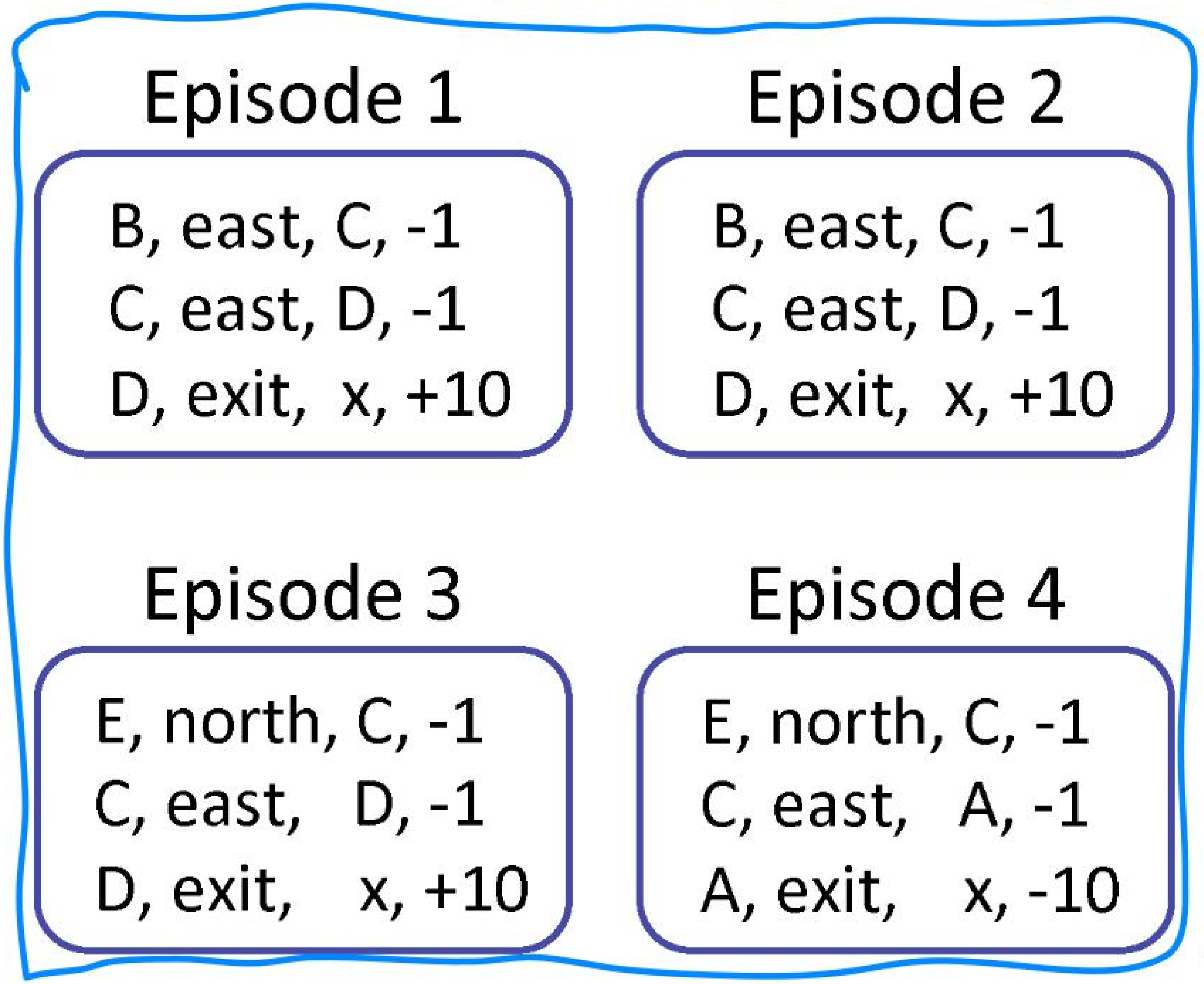

4

[4] Figure from [1]

Learning transition model

$p(\text{C} \mid \text{east}, \text{D}) = ?$

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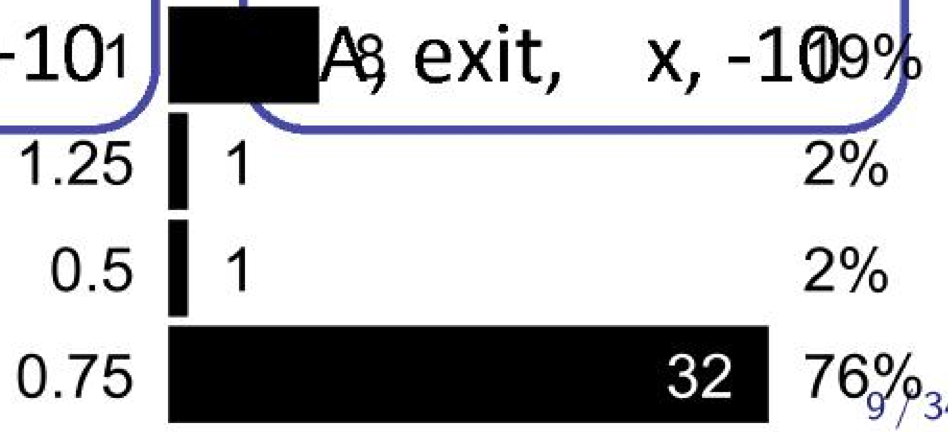
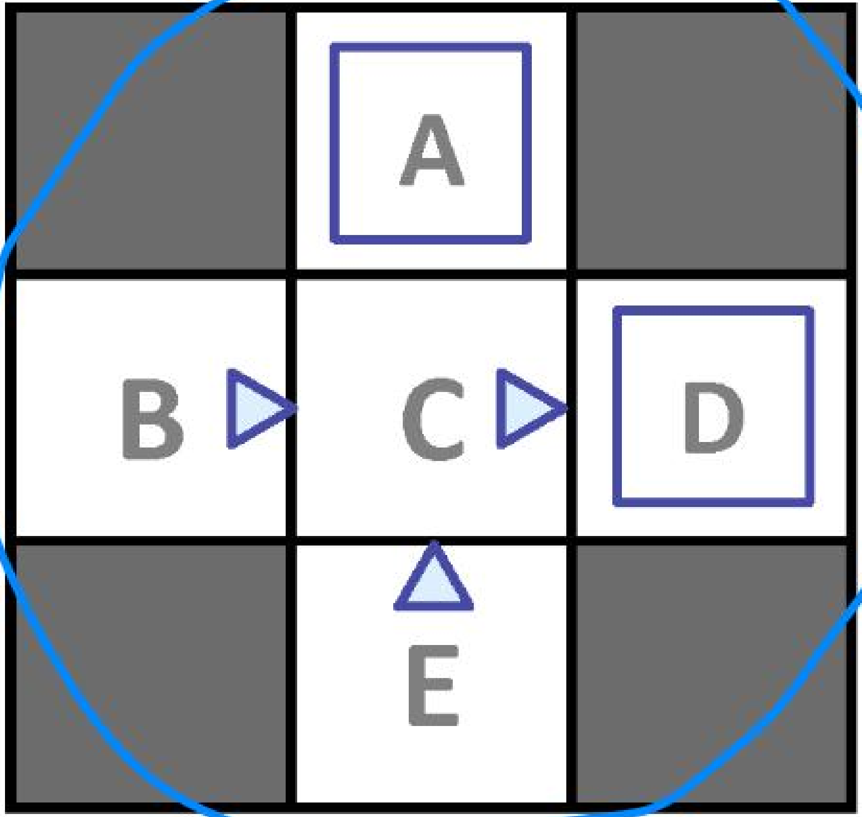


Model-based learning: Grid example

Input Policy π



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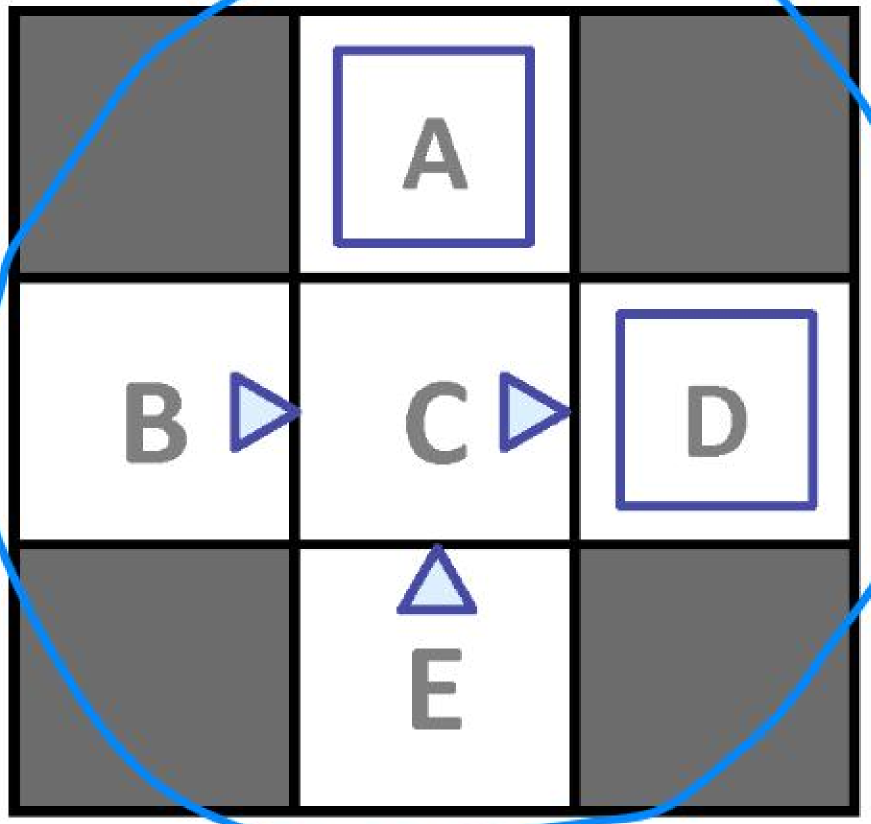
[4] Figure from [1]

Model-based learning: Grid example

Input Policy π



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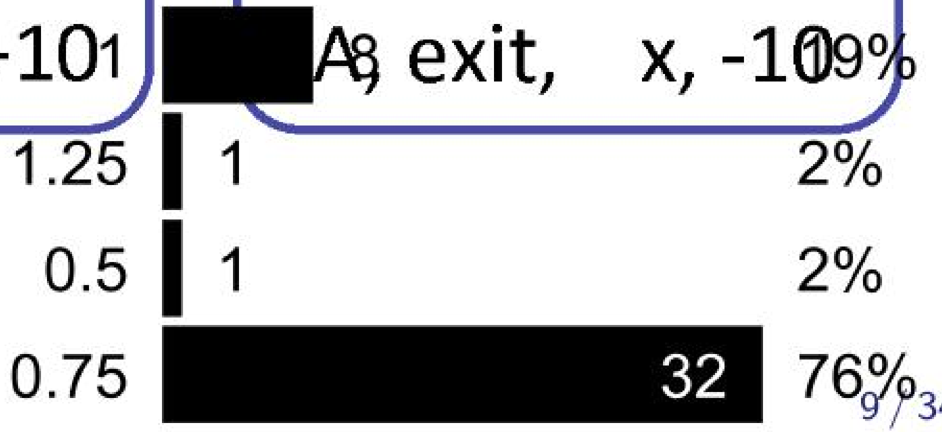

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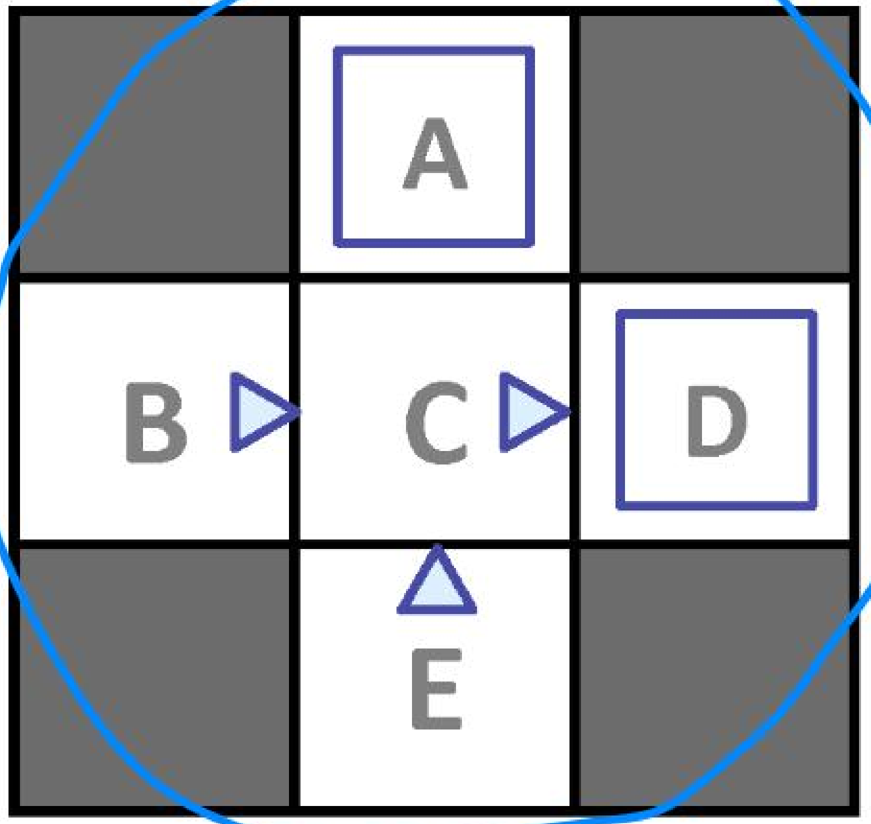

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Episode 3	Episode 4
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C, east, D, -1	C, east, A, -1
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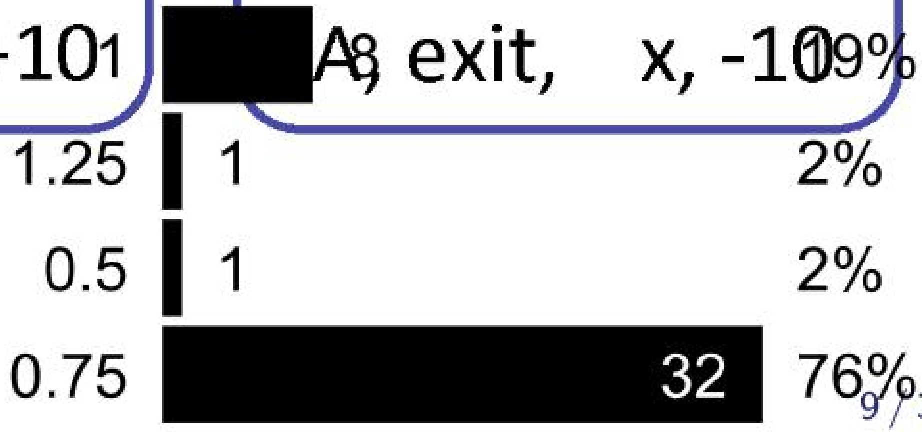

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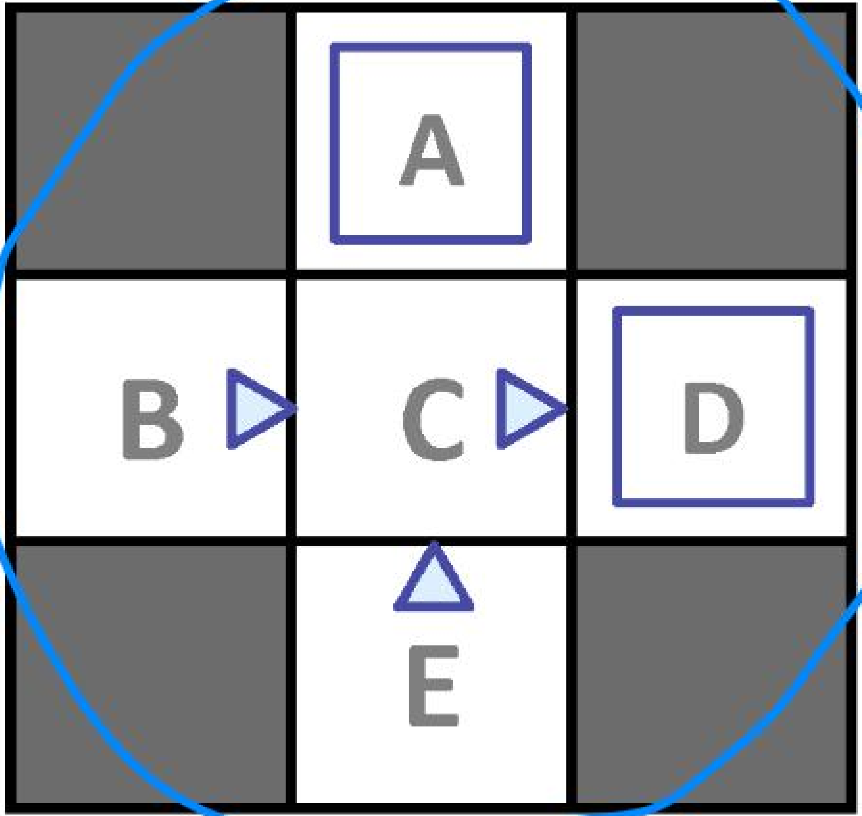

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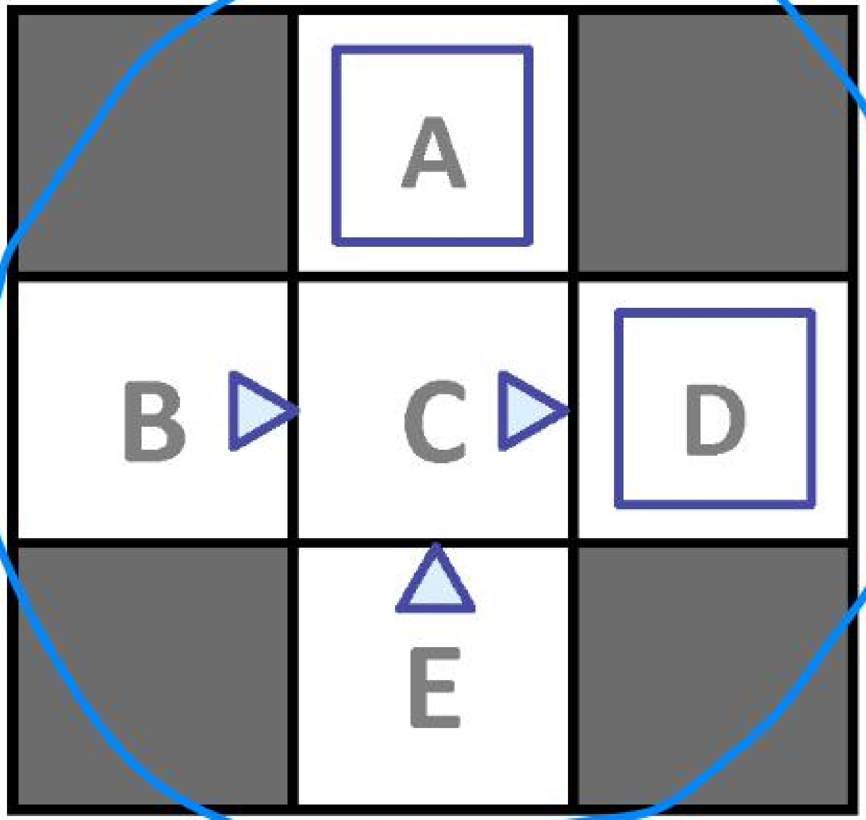

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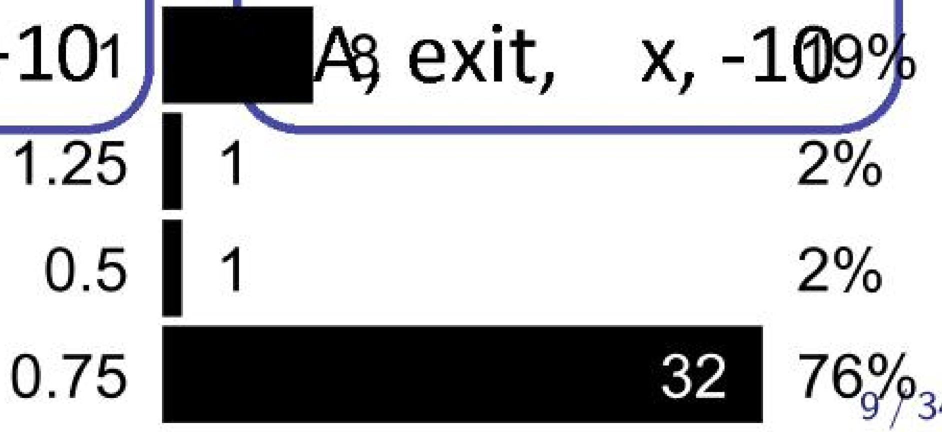

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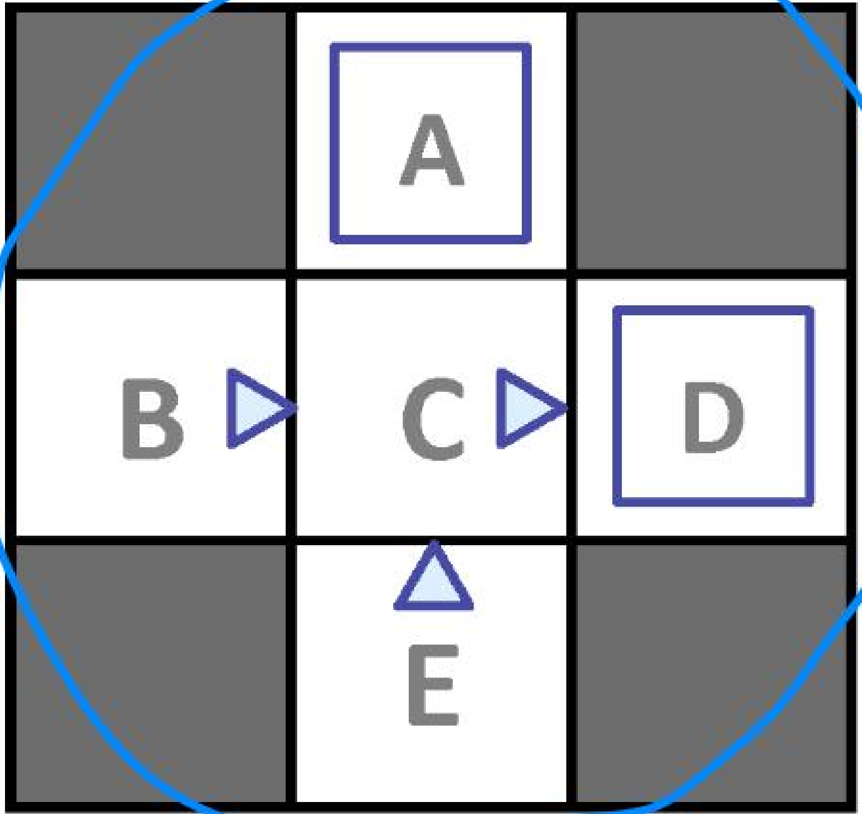

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Learning transition model

$p(D \mid east, C) = ?$

- $p(\mathrm{C} \mid \mathrm{east}, \mathrm{D}) = ?$

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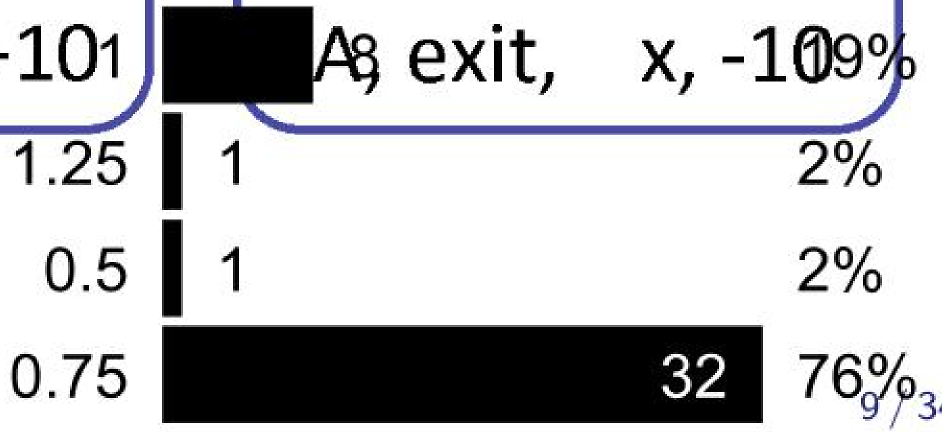

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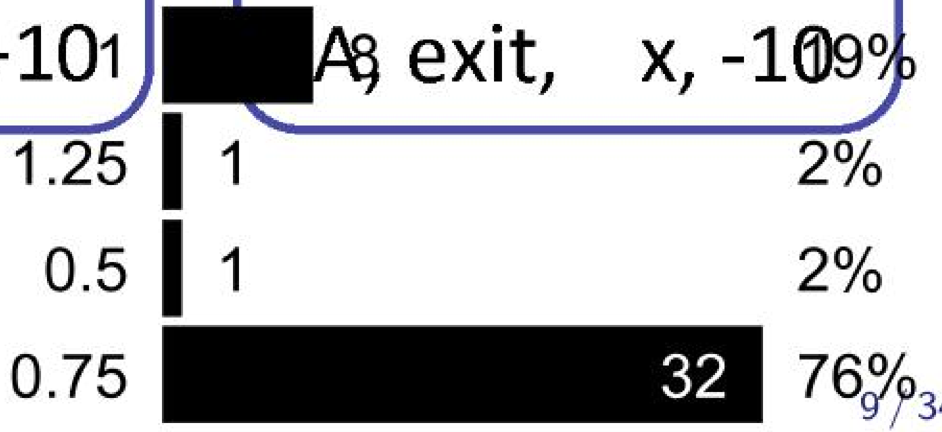

Learning reward function

$r(\mathrm{C}, \mathrm{east}, \mathrm{D}) = ?$

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Model based vs model-free: Expected age $\mathrm{E}[A]$

Random variable age A.

$$\mathrm{E}[A] = \sum_a P(A = a)a$$

We do not know $P(A = a)$, collecting N samples $[a_1, a_2, \dots a_N]$.

Model based

$$\hat{P}(a) = \frac{\text{num}(a)}{N}$$

$$\mathrm{E}[A] \approx \sum_a \hat{P}(a)a$$

Model free

$$\mathrm{E}[A] \approx \frac{1}{N} \sum_i a_i$$

Model based vs model-free: Expected age $\mathrm{E}[A]$

$a = \{18, 19, 20, \ldots, 30\}$

0.05 0.1 .

Random variable age A.

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N=1000

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$$\mathrm{E}[A] \approx \sum_a \hat{P}(a) a$$

$\hat{P}(a) \rightarrow P(a)$

Model free

$$\mathrm{E}[A] \approx \frac{1}{N} \sum_i a_i$$

Model-free learning

Passive learning

- **Input:** a fixed policy $\pi(s)$
- We want to know how good it is.
- r, p not known.
- Execute policy ...
- and learn on the way.
- **Goal:** learn the state values $v^{\pi}(s)$

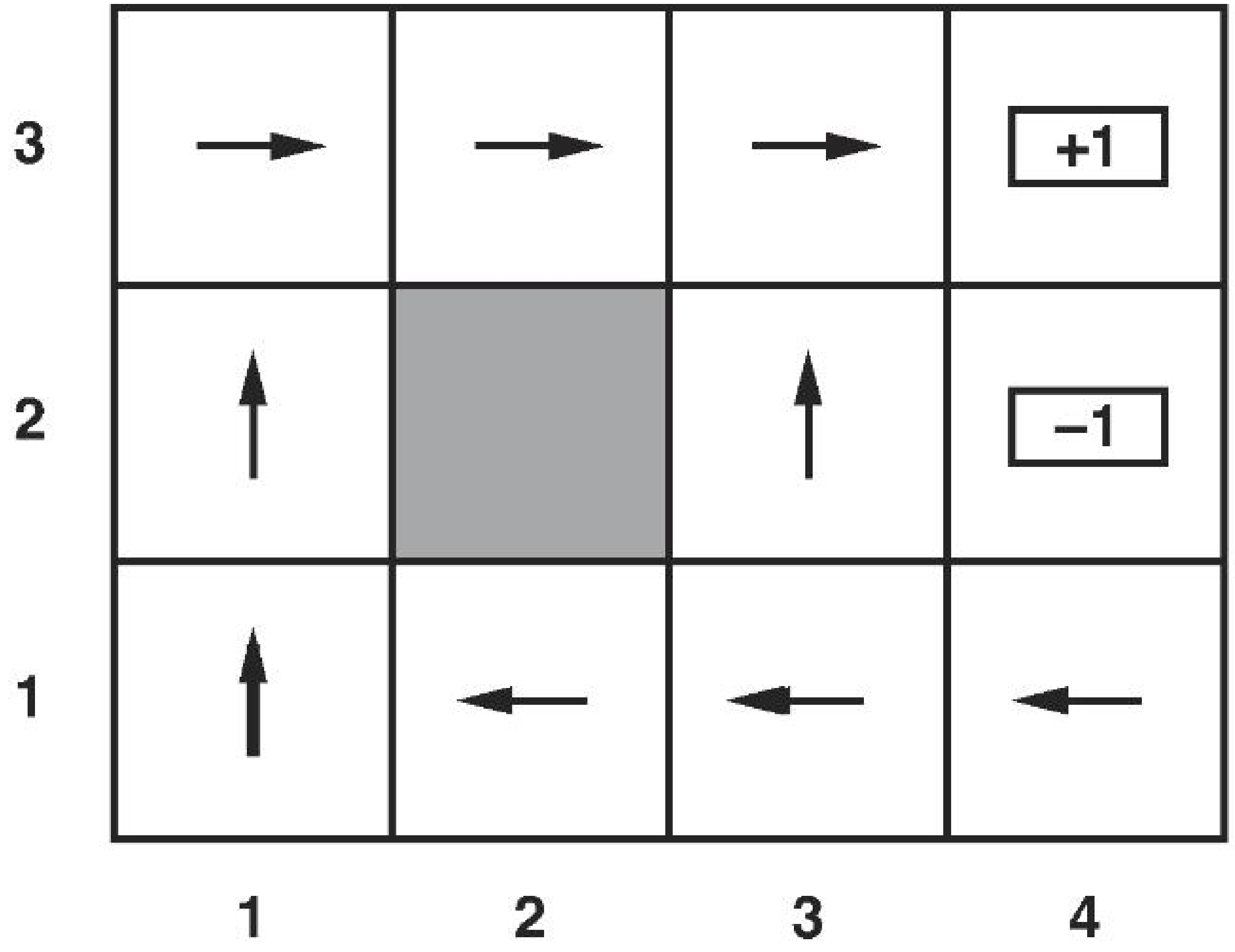

Image from [2]

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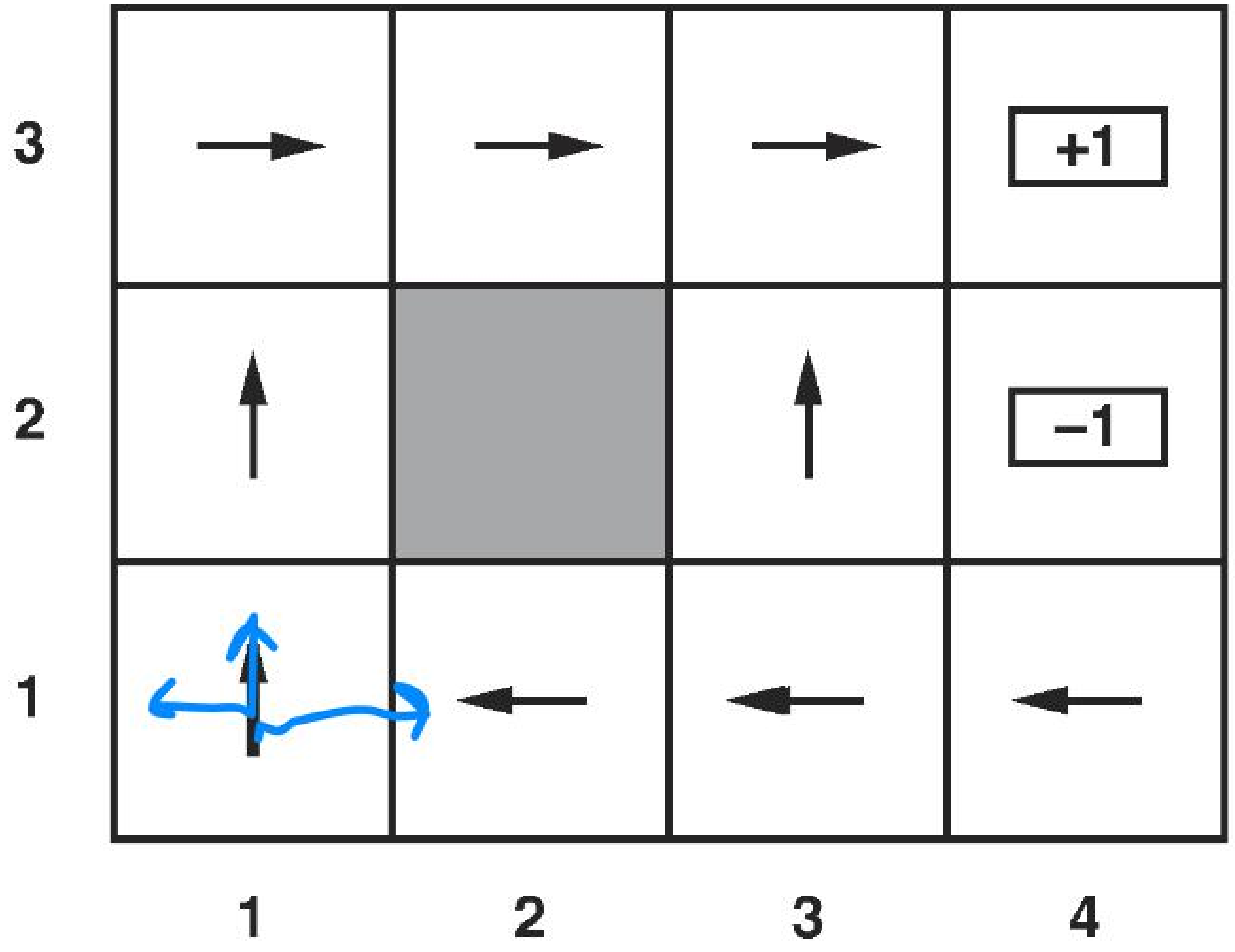


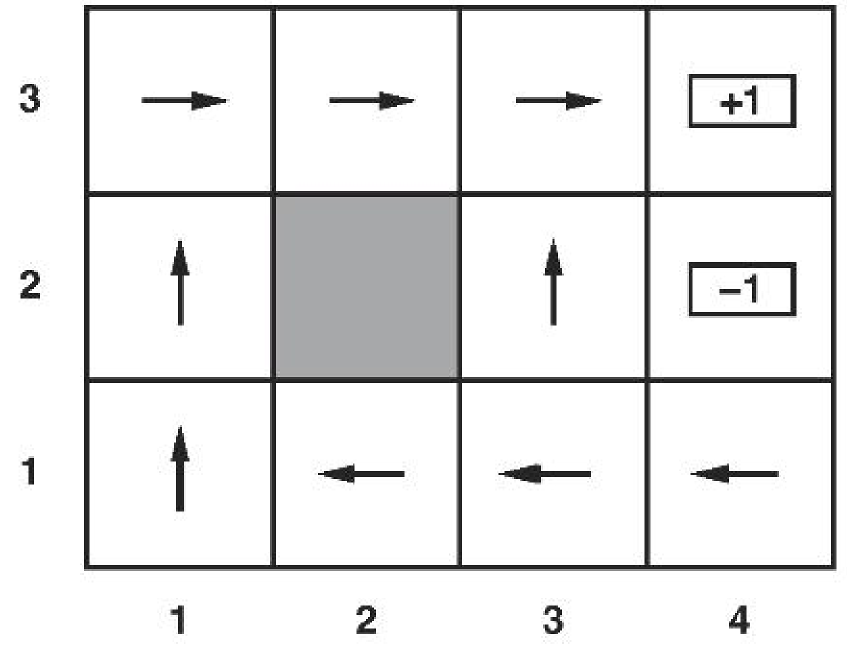
Image from [2]

Direct evaluation from episodes

Value of s for π – expected sum of discounted rewards – expected return

$$v^{\pi}(S_t) = \mathrm{E}\left[\sum_{k=0}^{\infty} \gamma^k R_{t+k+1}\right]$$

$$v^{\pi}(S_t) = \mathrm{E}\left[G_t\right]$$



Direct evaluation from episodes

Value of s for π – expected sum of discounted rewards – expected return

$$v^{\pi}(S_t) = \mathrm{E}\left[\sum_{k=0}^{\infty} \gamma^k R_{t+k+1}\right]$$

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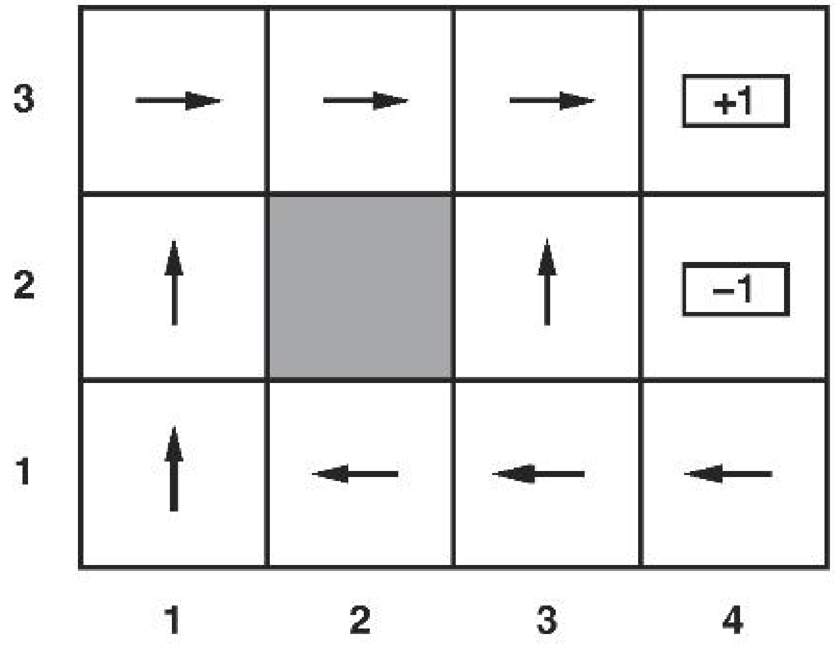

$$(1,1)_{\textbf{-.04}} \rightsquigarrow (1,2)_{\textbf{-.04}} \rightsquigarrow (1,3)_{\textbf{-.04}} \rightsquigarrow (1,2)_{\textbf{-.04}} \rightsquigarrow (1,3)_{\textbf{-.04}} \rightsquigarrow (2,3)_{\textbf{-.04}} \rightsquigarrow (3,3)_{\textbf{-.04}} \rightsquigarrow (4,3)_{\textbf{+1}}$$
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Direct evaluation from episodes

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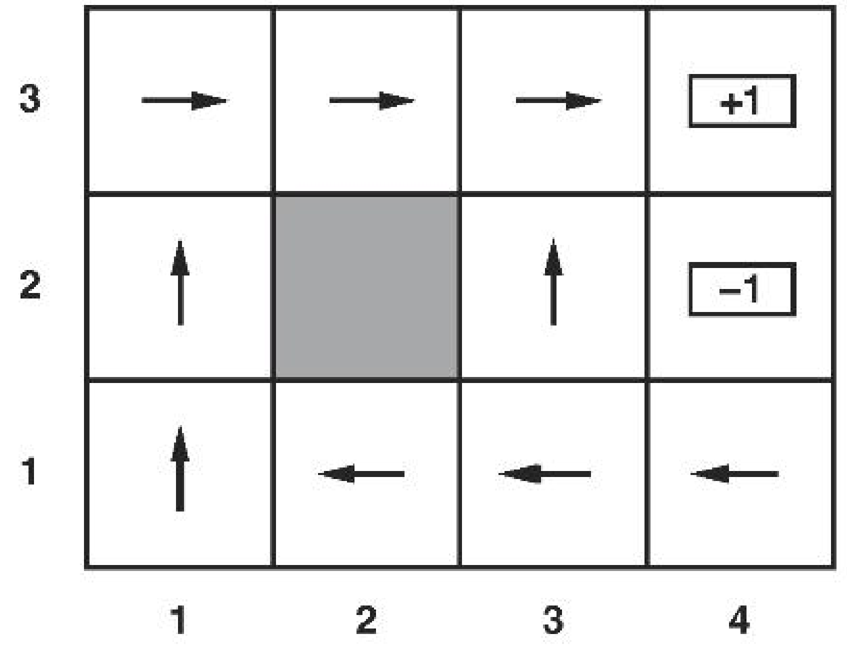

1. $(1,1)_{\mathbf{-.04}} \rightsquigarrow (1,2)_{\mathbf{-.04}} \rightsquigarrow (1,3)_{\mathbf{-.04}} \rightsquigarrow (1,2)_{\mathbf{-.04}} \rightsquigarrow (1,3)_{\mathbf{-.04}} \rightsquigarrow (2,3)_{\mathbf{-.04}} \rightsquigarrow (3,3)_{\mathbf{-.04}} \rightsquigarrow (4,3)_{\mathbf{+1}}$
2. $(1,1)_{\mathbf{-.04}} \rightsquigarrow (1,2)_{\mathbf{-.04}} \rightsquigarrow (1,3)_{\mathbf{-.04}} \rightsquigarrow (2,3)_{\mathbf{-.04}} \rightsquigarrow (3,3)_{\mathbf{-.04}} \rightsquigarrow (3,2)_{\mathbf{-.04}} \rightsquigarrow (3,3)_{\mathbf{-.04}} \rightsquigarrow (4,3)_{\mathbf{+1}}$
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Direct evaluation from episodes, $v^{\pi}(S_t) = \mathrm{E}[G_t]$, $\gamma = 1$

$$(1,1)_{\textbf{-.04}} \rightsquigarrow (1,2)_{\textbf{-.04}} \rightsquigarrow (1,3)_{\textbf{-.04}} \rightsquigarrow (1,2)_{\textbf{-.04}} \rightsquigarrow (1,3)_{\textbf{-.04}} \rightsquigarrow (2,3)_{\textbf{-.04}} \rightsquigarrow (3,3)_{\textbf{-.04}} \rightsquigarrow (4,3)_{\textbf{+1}}$$
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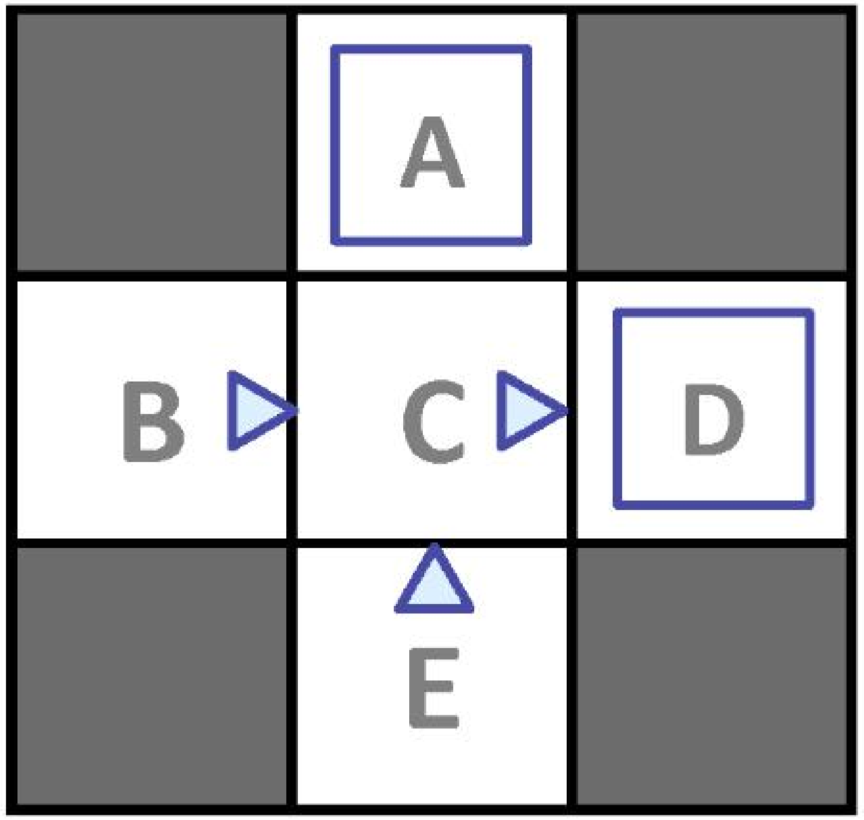
What is $v(3,2)$ after these episodes?

Direct evaluation: Grid example

Input Policy π



Assume: $\gamma = 1$

Observed Episodes (Training)

Episode 1

B, east, C, -1
C, east, D, -1
D, exit, x, +10

Episode 2

B, east, C, -1
C, east, D, -1
D, exit, x, +10

Episode 3

E, north, C, -1
C, east, D, -1
D, exit, x, +10

Episode 4

E, north, C, -1
C, east, A, -1
A, exit, x, -10

Direct evaluation: Grid example, $\gamma = 1$

What is v(C) after the 4 episodes?

Episode 1

B, east, C, -1
C, east, D, -1
D, exit, x, +10

Episode 2

B, east, C, -1
C, east, D, -1
D, exit, x, +10

Episode 3

E, north, C, -1
C, east, D, -1
D, exit, x, +10

Episode 4

E, north, C, -1
C, east, A, -1
A, exit, x, -10

Direct evaluation: Grid example, $\gamma = 1$

What is $v(\mathrm{C})$ after the 4 episodes?

$$\frac{9+9+9-11}{4} = 4$$

Episode 1

B, east, C, -1
C, east, D, -1
D, exit, x, +10

Episode 2

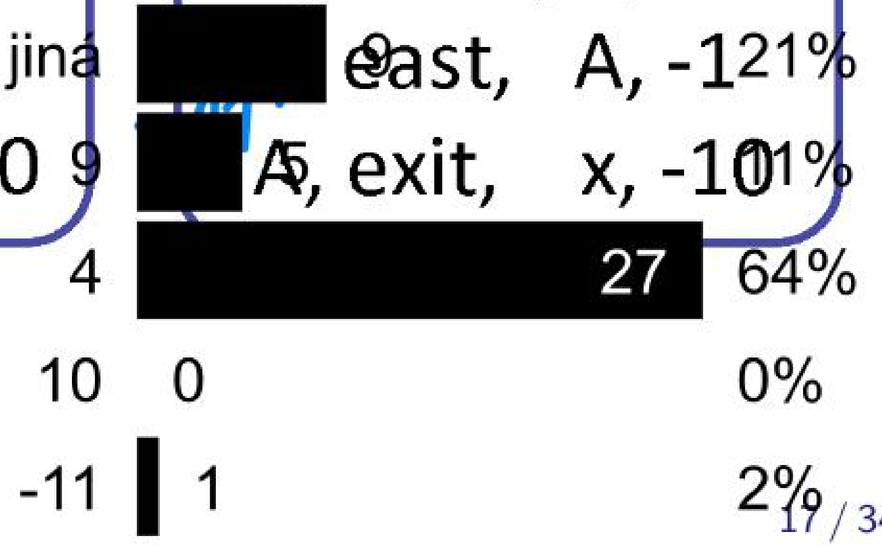
B, east, C, -1
C, east, D, -1
D, exit, x, +10

Episode 3

E, north, C, -1
C, east, D, -1
D, exit, x, +10

Episode 4

E, north, C, -1
east, A, -1
A, exit, x, -10



Direct evaluation algorithm

$(1,1)_{\textbf{-.04}} \rightsquigarrow (1,2)_{\textbf{-.04}} \rightsquigarrow (1,3)_{\textbf{-.04}} \rightsquigarrow (1,2)_{\textbf{-.04}} \rightsquigarrow (1,3)_{\textbf{-.04}} \rightsquigarrow (2,3)_{\textbf{-.04}} \rightsquigarrow (3,3)_{\textbf{-.04}} \rightsquigarrow (4,3)_{\textbf{+1}}$

$(1,1)_{\textbf{-.04}} \rightsquigarrow (1,2)_{\textbf{-.04}} \rightsquigarrow (1,3)_{\textbf{-.04}} \rightsquigarrow (2,3)_{\textbf{-.04}} \rightsquigarrow (3,3)_{\textbf{-.04}} \rightsquigarrow (3,2)_{\textbf{-.04}} \rightsquigarrow (3,3)_{\textbf{-.04}} \rightsquigarrow (4,3)_{\textbf{+1}}$

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Input: a policy π to be evaluated

Initialize:

    $V(s) \in \mathbb{R}$, arbitrarily, for all $s \in \mathcal{S}$
    $Returns(s) \leftarrow$ an empty list, for all $s \in \mathcal{S}$

Loop forever (for each episode):

    Generate an episode following π: $S_0, A_0, R_1, S_1, A_1, R_2, \ldots, S_{T-1}, A_{T-1}, R_T$
    $G \leftarrow 0$
    Loop for each step of episode, $t = T-1, T-2, \ldots, 0$:
        $G \leftarrow \gamma G + R_{t+1}$
        Unless S_t appears in $S_0, S_1, \ldots, S_{t-1}$:
            Append G to $Returns(S_t)$
            $V(S_t) \leftarrow \text{average}(Returns(S_t))$

Direct evaluation algorithm

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      $V(S_t) \leftarrow \text{average}(Returns(S_t))$

v(s)
q(s, a)

^C

Direct evaluation: analysis

The good:

- Simple, easy to understand and implement.
- Does not need p, r and eventually it computes the true v^π.

The bad:

- Each state value learned in isolation.
- State values are not independent
- $v^\pi(s) = \sum_{s'} p(s' \mid s, \pi(s))[r(s, \pi(s), s') + \gamma v^\pi(s')]$

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$$(1,1)_{-.04} \rightsquigarrow (1,2)_{-.04} \rightsquigarrow (1,3)_{-.04} \rightsquigarrow (1,2)_{-.04} \rightsquigarrow (1,3)_{-.04} \rightsquigarrow (2,3)_{-.04} \rightsquigarrow (3,3)_{-.04} \rightsquigarrow (4,3)_{+1}$$
$$(1,1)_{-.04} \rightsquigarrow (1,2)_{-.04} \rightsquigarrow (1,3)_{-.04} \rightsquigarrow (2,3)_{-.04} \rightsquigarrow (3,3)_{-.04} \rightsquigarrow (3,2)_{-.04} \rightsquigarrow (3,3)_{-.04} \rightsquigarrow (4,3)_{+1}$$
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Policy evaluation?

In each round, replace V with a one-step-look-ahead

$V_0^\pi(s) = 0$

$V_{k+1}^\pi(s) \leftarrow \sum_{s'} p\big(s' \mid s, \pi(s)\big)\big[r(s, \pi(s), s') + \gamma\, V_k^\pi(s')\big]$

Problem: both $p\big(s' \mid s, \pi(s)\big)$ and $r(s, \pi(s), s')$ unknown!

Policy evaluation?

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Problem: both $p\big(s' \mid s, \pi(s)\big)$ and $r(s, \pi(s), s')$ unknown!

Use samples for evaluating policy?

MDP (p, r known) : Update V estimate by a weighted average:
$V_{k+1}^{\pi}(s) \leftarrow \sum_{s'} p\big(s' \mid s, \pi(s)\big)\big[r(s, \pi(s), s') + \gamma\, V_k^{\pi}(s')\big]$

What about try and average? Trials at time t

$$\begin{aligned} \text{trial}^1 &= R_{t+1}^1 + \gamma\, V(S_{t+1}^1) \\ \text{trial}^2 &= R_{t+1}^2 + \gamma\, V(S_{t+1}^2) \\ \vdots &= \vdots \\ \text{trial}^n &= R_{t+1}^n + \gamma\, V(S_{t+1}^n) \end{aligned}$$

$$V(S_t) \leftarrow \frac{1}{n} \sum_i \text{trial}^i$$

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$V^{\pi}_{k+1}(s) \leftarrow \sum_{s'} p\big(s' \mid s, \pi(s)\big)\big[r(s, \pi(s), s') + \gamma\, V^{\pi}_{k}(s')\big]$

What about try and average? Trials at time t

$$
\begin{aligned}
\text{trial}^1 &= R^1_{t+1} + \gamma\, V(S^1_{t+1}) \\
\text{trial}^2 &= R^2_{t+1} + \gamma\, V(S^2_{t+1}) \\
\vdots &= \vdots \\
\text{trial}^n &= R^n_{t+1} + \gamma\, V(S^n_{t+1})
\end{aligned}
$$

$$
V(S_t) \leftarrow \frac{1}{n} \sum_i \text{trial}^i
$$

Temporal-difference value learning

$$(1,1)_{\textbf{-.04}} \rightsquigarrow (1,2)_{\textbf{-.04}} \rightsquigarrow (1,3)_{\textbf{-.04}} \rightsquigarrow (1,2)_{\textbf{-.04}} \rightsquigarrow (1,3)_{\textbf{-.04}} \rightsquigarrow (2,3)_{\textbf{-.04}} \rightsquigarrow (3,3)_{\textbf{-.04}} \rightsquigarrow (4,3)_{\textbf{+1}}$$
$$(1,1)_{\textbf{-.04}} \rightsquigarrow (1,2)_{\textbf{-.04}} \rightsquigarrow (1,3)_{\textbf{-.04}} \rightsquigarrow (2,3)_{\textbf{-.04}} \rightsquigarrow (3,3)_{\textbf{-.04}} \rightsquigarrow (3,2)_{\textbf{-.04}} \rightsquigarrow (3,3)_{\textbf{-.04}} \rightsquigarrow (4,3)_{\textbf{+1}}$$
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$\gamma = 1$

From first trial (episode): $V(2,3) = 0.92$, $V(1,3) = 0.84, \ldots$

In second episode, going from $S_t = (1,3)$ to $S_{t+1} = (2,3)$ with reward $R_{t+1} = -0.04$, hence:

$$V(1,3) = R_{t+1} + V(2,3) = -0.04 + 0.92 = 0.88$$

- First estimate 0.84 is a bit lower than 0.88. $V(S_t)$ is different than $R_{t+1} + \gamma V(S_{t+1})$
- Update: $V(S_t) \leftarrow V(S_t) + \alpha\Big(\big[R_{t+1} + \gamma V(S_{t+1})\big] - V(S_t)\Big)$
- α is the learning rate.
- $V(S_t) \leftarrow (1-\alpha)V(S_t) + \alpha\,$(new sample)

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\text{trial}^n &= R^n_{t+1} + \gamma\, V(S^n_{t+1})
\end{aligned}
$$

$$
V(S_t) \leftarrow \frac{1}{n} \sum_i \text{trial}^i
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Temporal-difference value learning

$$(1,1)_{\text{-.04}} \rightsquigarrow (1,2)_{\text{-.04}} \rightsquigarrow (1,3)_{\text{-.04}} \rightsquigarrow (1,2)_{\text{-.04}} \rightsquigarrow (1,3)_{\text{-.04}} \rightsquigarrow (2,3)_{\text{-.04}} \rightsquigarrow (3,3)_{\text{-.04}} \rightsquigarrow (4,3)_{\text{+1}}$$
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Temporal-difference value learning

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Exponential moving average

$$\overline{x}_n = (1 - \alpha)\overline{x}_{n-1} + \alpha x_n$$

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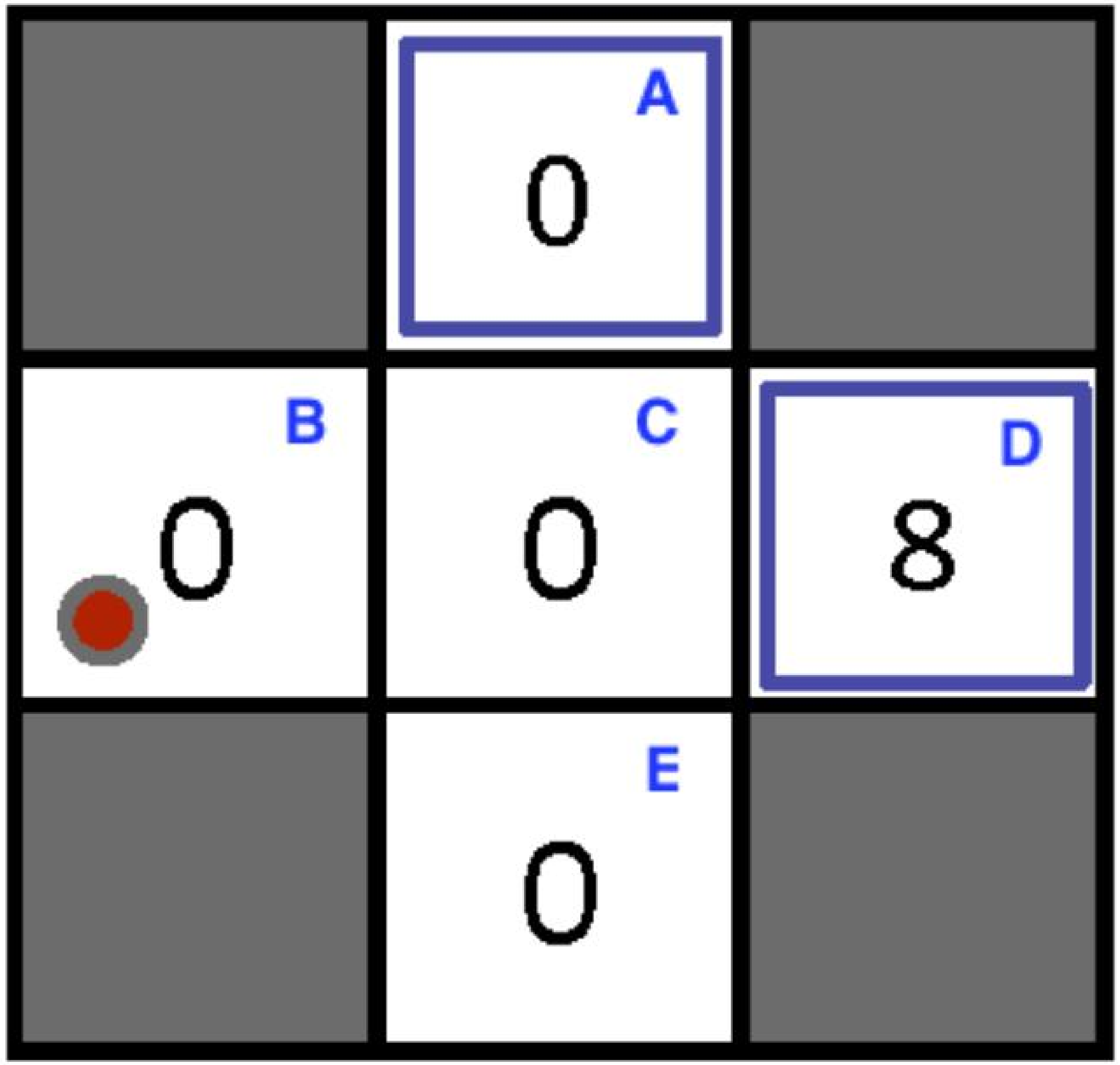
Example: TD Value learning

$$V(S_t) \leftarrow V(S_t) + \alpha(R_{t+1} + \gamma V(S_{t+1}) - V(S_t))$$



- Values represent initial $V(s)$
- Assume: $\gamma = 1, \alpha = 0.5, \pi(s) = \rightarrow$
- $(B, \rightarrow, C), -2, \Rightarrow V(B)?$
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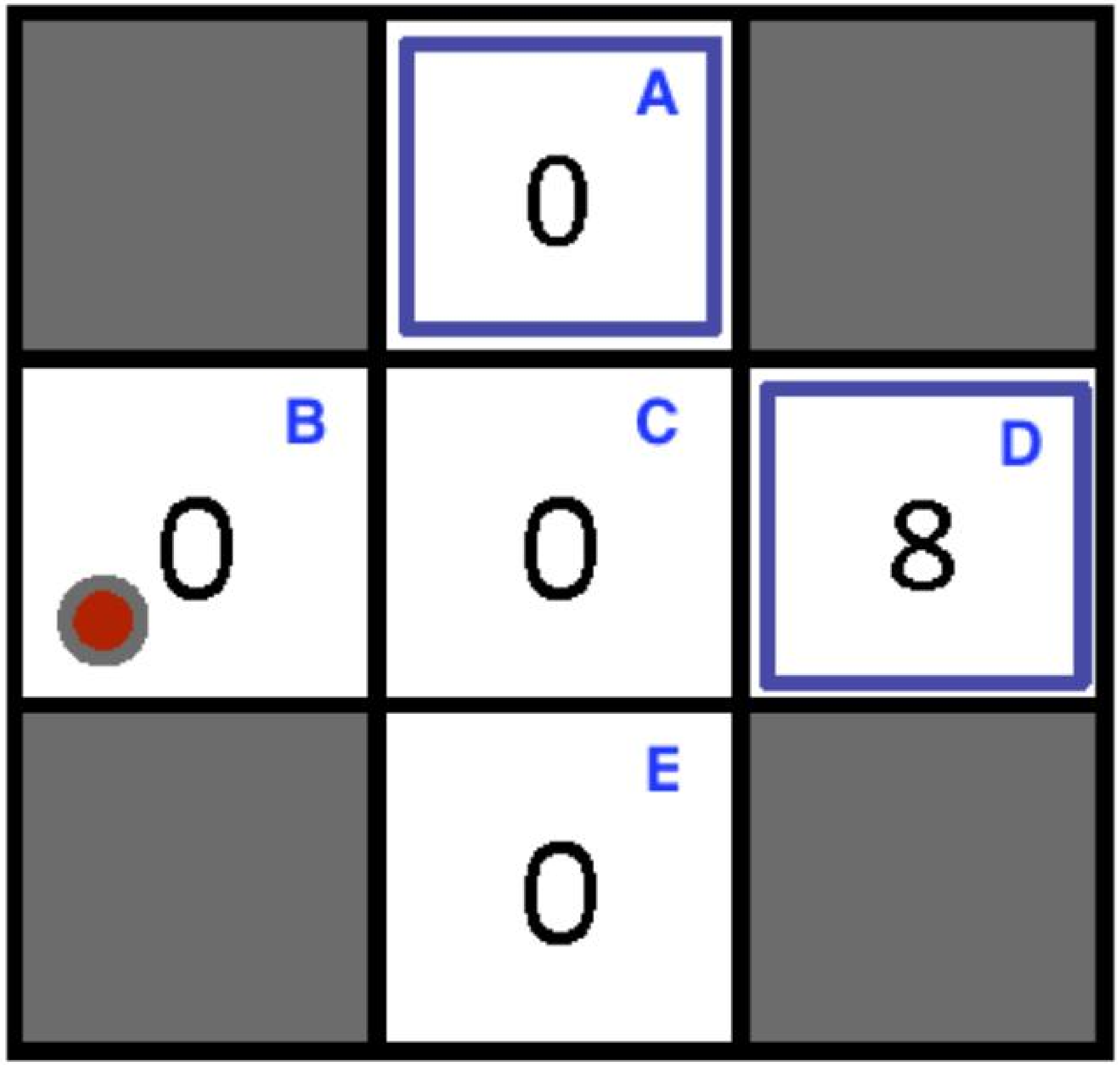

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$\emptyset \quad \tfrac{1}{2}(-2 + 1.\emptyset - \emptyset) = -1$

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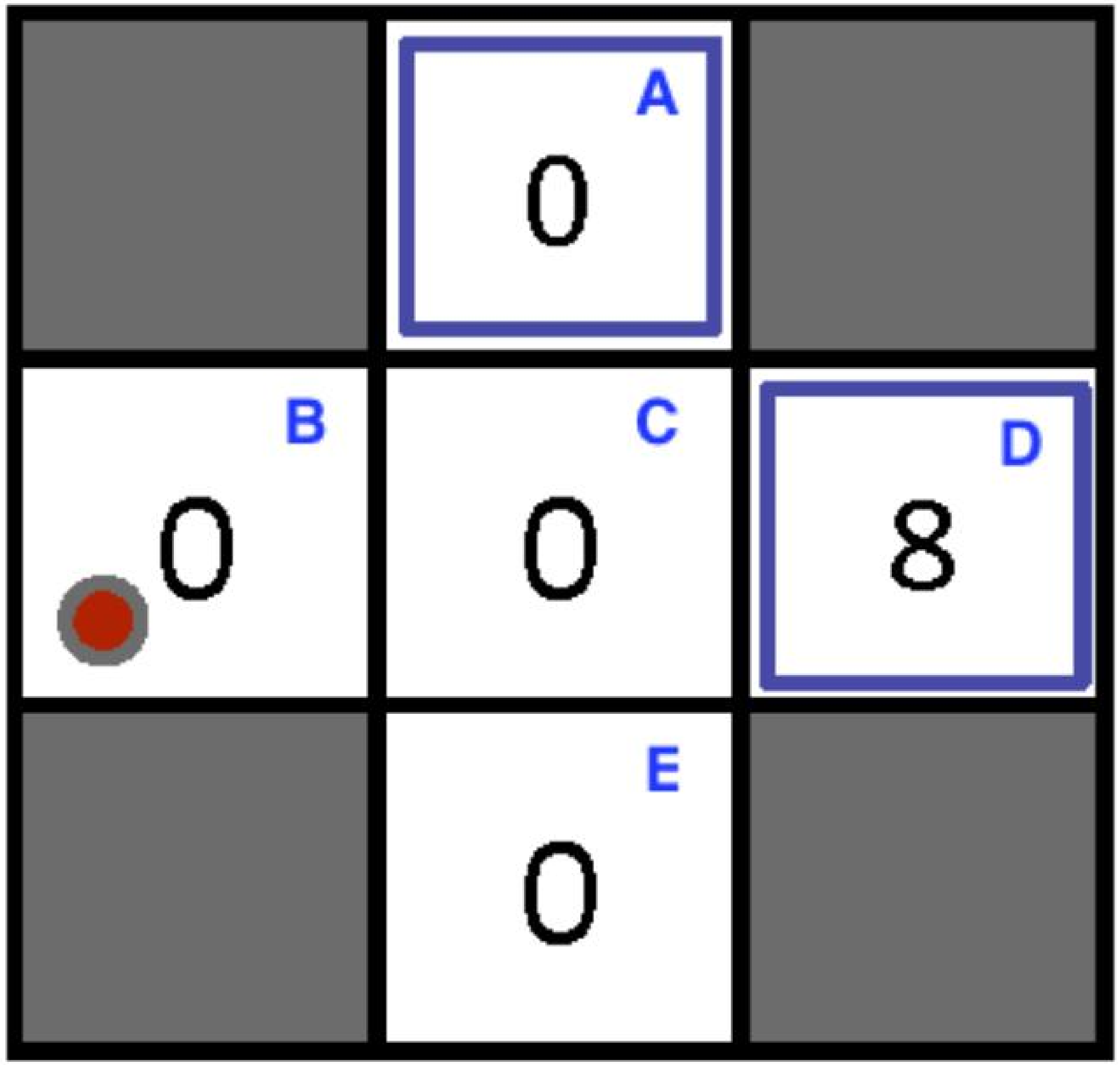

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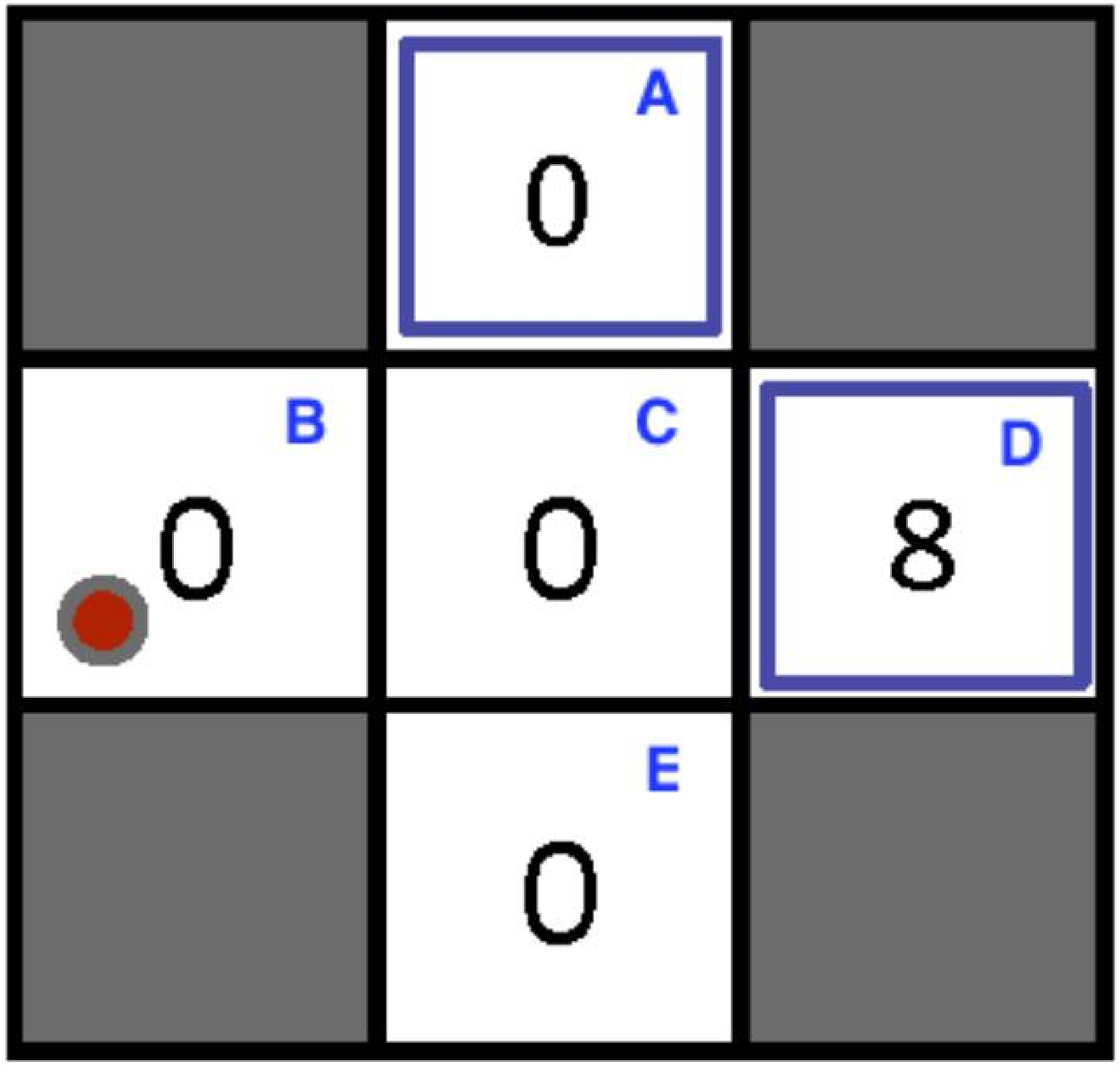

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Temporal difference value learning: algorithm

Input: the policy π to be evaluated
Algorithm parameter: step size $\alpha \in (0, 1]$
Initialize $V(s)$, for all $s \in \mathcal{S}^+$, arbitrarily except that $V(terminal) = 0$

Loop for each episode:
  Initialize S
  Loop for each step of episode:
    $A \leftarrow$ action given by π for S
    Take action A, observe R, S'
    $V(S) \leftarrow V(S) + \alpha\big[R + \gamma V(S') - V(S)\big]$
    $S \leftarrow S'$
  until S is terminal

What is wrong with the temporal difference Value learning?

The Good: Model-free value learning through mimicking Bellman updates

The Bad: How to turn values into a (new) policy?

- $\pi(s) = \arg\max_{a} \sum_{s'} p(s' \mid s, a) \left[r(s, a, s') + \gamma V(s') \right]$

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Active reinforcement learning

Reminder: V, Q-value iteration for MDPs

Value/Utility iteration (depth limited evaluation):

- Start: $V_0(s) = 0$
- In each step update V by looking one step ahead:
  $V_{k+1}(s) \leftarrow \max_a \sum_{s'} p(s' \mid s, a) \left[r(s, a, s') + \gamma V_k(s')\right]$

Q values more useful (think about updating π)

- Start: $Q_0(s, a) = 0$
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Learn Q values as the robot/agent goes (temporal difference)

- Drive the robot and fetch rewards (s, a, s', R)
- We know old estimates $Q(s,a)$ (and $Q(s',a')$), if not, initialize.
- A new trial/sample estimate at time t
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- α update
  $Q(S_t, A_t) \leftarrow Q(S_t, A_t) + \alpha(\text{trial} - Q(S_t, A_t))$
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In each step Q approximates the optimal q^* function.

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MDP update: $Q_{k+1}(s,a) \leftarrow \sum_{s'} p(s' \mid s,a) \left[r(s,a,s') + \gamma \max_{a'} Q_k(s',a') \right]$

Learn Q values as the robot/agent goes (temporal difference)

- Drive the robot and fetch rewards (s, a, s', R)
- We know old estimates $Q(s,a)$ (and $Q(s',a')$), if not, initialize.
- A new trial/sample estimate at time t
  trial $= R_{t+1} + \gamma \max_a Q(S_{t+1}, a)$
- α update
  $Q(S_t, A_t) \leftarrow Q(S_t, A_t) + \alpha(\text{trial} - Q(S_t, A_t))$
  $Q(S_t, A_t) \leftarrow (1-\alpha)Q(S_t, A_t) + \alpha\,\text{trial}$

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